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Patnaiks' Theory of Imperialism: A Ricardo's Fallacy?*

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Abstract

We consider a crucial aspect of the theory of imperialism developed in a complete way by Patnaik and Patnaik (2017), namely, the asserted failure of the Ricardian theory of comparative advantage when there is a "material asymmetry" due to the fact that some goods are produced almost exclusively in the so-called "global south." This point was already raised by Patnaik (2005) as a "Ricardo's fallacy." We address the issue by providing an analytical specification of the Ricardo model of international trade with two goods, two countries, and with Mill-Graham preferences, represented by a Cobb-Douglas utility function. In this structure, we show that there are gains from international trade despite of the material asymmetry and consequently there is no Ricardo's fallacy.

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1 Introduction

Patnaik and Patnaik (2017) introduced a new theory of imperialism, based on the idea of a "material asymmetry" between the so-called "global north" and "global south." According to these authors, the material asymmetry consists in the fact that some "tropical goods" are not produced at all, or they are produced in insufficient quantity, in the global north, because of climatic or other natural reasons.

One of the cornerstones of the Patnaiks new theory of imperialism is a criticism to the "mainstream trade theory" – identified with the traditional Ricardian theory of comparative advantage – as an implication of the material asymmetry. Indeed, the criticism to the Ricardian theory developed by Patnaik and Patnaik (2017) was based on the idea, already proposed by Patnaik (2005), of a presumed "Ricardo's fallacy."

Patnaik (2005) indeed claimed that "the Ricardian theory of comparative advantage contains a logical fallacy when used to argue that mutual benefit necessarily results from trade. Ricardo's two-country, two-commodity model assumes that both goods can be produced in both countries." (see p. 31). More precisely, she affirmed that "Ricardo's process of reasoning is valid but a material fallacy arises because his assumption or premise is not true for a general theory of trade. There is a serious problem with the assumption, which is that goods are producible and indeed are actually produced in both countries." (see p. 32).

Patnaik (2005) compared, through an example, the classical Ricardian case where each country can in principle produce both goods with a different case where one country can produce only one good whereas the other produces both goods. Her example confirmed that, in the first case, when each country only produces the good for which it has a comparative advantage, there are gains from trade, whereas, in the second case, even if the maximal amount of the two goods is produced, there is no gain from trade.

Patanik and Patnaik (2017) go further arguing: "The case for free trade is made nowadays on the basis of the argument that the "utility possibility curve" after trade in each country lies outside [...] the "utility possibility curve" before trade. For the "utility possibility curves" to have this relationship, the bundle of goods available to each country must be vector-wise larger with trade than without trade [...]." (see p. 13). Clearly, if one of the two countries produces only one of the two goods, it must have less of that good after trade in order to have a positive amount of the good it does not produce. Therefore, according to Patnaik and Patnaik (2017), there cannot be gains from trade as the bundle of goods available to that country cannot be vector-wise larger with trade than without trade.

It must be stressed that the argument provided by the Patnaiks is implicitly based on the assumption that preferences for goods in both countries are strongly monotone. Indeed, under this assumption, a vector-wise increase in the bundle of goods available to each country after international trade is a sufficient condition for gains from trade. Nevertheless, we shall show that this condition is not also necessary.

In this paper, we first provide a formal specification of the Ricardo model with two goods, two countries, and Mill-Graham preferences, represented by a Cobb-Douglas utility function with equal expenditure shares, both under the assumption of autarky and that of international trade. In particular, we fully characterize the situations under which there are gains from international trade. Then, we reconsider the asymmetry introduced by Patanik (2005) and Patnaik and Patnaik (2017) assuming that one of the two goods is not produced in one of the two countries. We show that, in this case, there are still gains from trade contradicting the thesis of a Ricardo fallacy claimed in the two articles by the Patnaiks.

The paper is organized as follows. In Section 2, we specify the Ricardo model with Cobb-Douglas preferences. In Section 3, we analyze the Ricardo fallacy. In Section 4, we conclude.

2 The Ricardo model of trade with Cobb-Douglas preferences

In this section, we present a reformulation of the basic Ricardo model of trade which takes inspiration from the standard textbook version proposed by Feenstra (2003).

There are two countries indexed by the subscript j, $j = 1, 2$, which produce two goods indexed by the subscript i, $i = 1, 2$. The two countries produce the two goods in two industries using labor L as the unique factor of production. We denote by $\bar{L}_j$ the total labor force in country j, $j = 1, 2$. Moreover, we denote by a_{ij} the labor needed to produce one unit of good i in country j, $i = 1, 2$, $j = 1, 2$.

We make the following two assumptions, typical of the Ricardo model.

Assumption 1. *Labor is perfectly mobile between the industries in each country and immobile across countries.*

Assumption 2. $0 < a_{ij} < +\infty$, $i = 1, 2$, $j = 1, 2$.

Good i is produced in country j using a production function $f_{ij}(\cdot)$ such that $f_{ij}(L) = \frac{L}{a_{ij}}$, $i = 1, 2$, $j = 1, 2$. Let y_{ij} be the quantity of good i produced in country j, $i = 1, 2$, $j = 1, 2$. Then, the production possibility frontier of country j is given by the following equation

$$a_{1j}y_{1j} + a_{2j}y_{2j} = \bar{L}_j,$$

$j = 1, 2$. We denote by p_{ij} the price of good i in country j and by w_{ij} the wage earned in the industry which produces good i in country j, $i = 1, 2$, $j = 1, 2$. Moreover, we denote by $\pi_{ij}(L)$ the profit function of industry i in country j, $i = 1, 2$, $j = 1, 2$. Clearly, it must be that

$$\pi_{ij}(L) = p_{ij}f_{ij}(L) - w_{ij}L = p_{ij}\frac{L}{a_{ij}} - w_{ij}L,$$

$i = 1, 2$, $j = 1, 2$.

We introduce now a further standard Ricardian assumption.

Assumption 3. *There is perfect competition in goods and labor markets in each country.*

The following proposition is a straightforward consequence of Assumption 1.

Proposition 1. *Under Assumption 1, if* $w_{1j} > w_{2j}$, *then* $y_{1j} = \frac{\bar{L}_j}{a_{1j}}$ *and* $y_{2j} = 0$*; if* $w_{1j} < w_{2j}$, *then* $y_{1j} = 0$ *and* $y_{2j} = \frac{\bar{L}_j}{a_{2j}}$, $j = 1, 2$.

The following proposition is a straightforward consequence of Assumption 3.

Proposition 2. *Under Assumption 3,* $\pi_{ij}(L) = 0$, $i = 1, 2$, $j = 1, 2$.

The next proposition characterizes prices and wages in autarky, i.e., when there is not international trade.

Proposition 3. *Under Assumptions 1, 2, and 3, in autarky,* $w_{1j} = w_j = w_{2j}$ *and* $\frac{p_{1j}}{p_{2j}} = \frac{a_{1j}}{a_{2j}}$, $j = 1, 2$.

The following assumption states that preferences for the two goods in the two countries are the same, of the Mill-Graham type, and represented by Cobb-Douglas utility functions with equal expenditure shares.

Assumption 4. $u_j(x_{1j}, x_{2j}) = x_{1j}x_{2j}$, $j = 1, 2$.

Let $x_{1j}(p_{1j}, p_{2j})$ and $x_{2j}(p_{1j}, p_{2j})$ be the demand, respectively, for good 1 and good 2 in country j, $j = 1, 2$.

An autarky equilibrium consists of a price $\tilde{p}_{ij}$ of good i in country j, a quantity $\tilde{x}_{ij}$ of good i demanded in country j, and a quantity $\tilde{y}_{ij}$ of good i supplied (produced) in country j, such that $\frac{\tilde{p}_{1j}}{\tilde{p}_{2j}} = \frac{a_{1j}}{a_{2j}}$, $\tilde{x}_{1j} = x_{1j}(\tilde{p}_{1j}, \tilde{p}_{2j}) = \tilde{y}_{1j}$ and $\tilde{x}_{2j} = x_{2j}(\tilde{p}_{1j}, \tilde{p}_{2j}) = \tilde{y}_{2j}$, $i = 1, 2$, $j = 1, 2$.

The following proposition provides the computation of the autarky equilibrium for our version of the Ricardo model.

Proposition 4. *Under Assumptions 1, 2, 3, and 4, at the autarky equilibrium, the quantities demanded and produced are* $\tilde{x}_{1j} = \frac{\bar{L}_j}{2a_{1j}} = \tilde{y}_{1j}$ *and* $\tilde{x}_{2j} = \frac{\bar{L}_j}{2a_{2j}} = \tilde{y}_{2j}$, $j = 1, 2$.

The next assumption states that country 1 has a comparative advantage in the production of good 1.

Assumption 5. $\frac{a_{11}}{a_{21}} < \frac{a_{12}}{a_{22}}$.

International trade is characterized as follows.

Assumption 6. *International trade between countries* 1 *and* 2 *is free and frictionless.*

We denote by p_i the international price of good i, $i = 1, 2$.

The next proposition is an immediate consequence of Assumption 6, which rules out any possibility of arbitrage between the two countries.

Proposition 5. *Under Assumption 6, with international trade,* $p_{11} = p_1 = p_{12}$ *and* $p_{21} = p_2 = p_{22}$.

We denote by $S(p_1, p_2)$ the international relative supply of good 1 with respect to good 2. Clearly, we have that

$$S(p_1, p_2) = \frac{y_{11} + y_{12}}{y_{21} + y_{22}}.$$

The following proposition provides a full specification of the international relative supply of good 1 with respect to good 2.

Proposition 6. *Under Assumptions 1, 2, 3, and 5, if* $\frac{p_1}{p_2} < \frac{a_{11}}{a_{21}}$, *then* $S(p_1, p_2) = 0$; *if* $\frac{p_1}{p_2} = \frac{a_{11}}{a_{21}}$, *then* $S(p_1, p_2) \in [0, (\bar{L}_1/a_{11})/(\bar{L}_2/a_{22})]$; *if* $\frac{a_{11}}{a_{21}} <$

$\frac{p_1}{p_2} < \frac{a_{12}}{a_{22}}$, *then* $S(p_1, p_2) = (\bar{L}_1/a_{11})/(\bar{L}_2/a_{22})$; *if* $\frac{p_1}{p_2} = \frac{a_{12}}{a_{22}}$, *then* $S(p_1, p_2) \in [(\bar{L}_1/a_{11})/(\bar{L}_2/a_{22}), +\infty]$.

We denote by $D(p_1, p_2)$ the international relative demand of good 1 with respect to good 2. Clearly, we have that

$$D(p_1, p_2) = \frac{x_{11}(p_1, p_2) + x_{12}(p_1, p_2)}{x_{21}(p_1, p_2) + x_{22}(p_1, p_2)}.$$

The next proposition is an immediate consequence of Assumption 4.

Proposition 7. *Under Assumption 4,* $D(p_1, p_2) = \frac{p_2}{p_1}$.

An international equilibrium consists of an international price p_i^* of good i, $i = 1, 2$, such that $D(p_1^*, p_2^*) = S(p_1^*, p_2^*)$.

The following proposition establishes a lower bound for the international equilibrium relative price of good 1 with respect to good 2.

Proposition 8. *Under Assumptions 1, 2, 3, 4, and 5,* $\frac{p_1^*}{p_2^*} \geq \frac{a_{11}}{a_{21}}$.

Proof. Suppose that $\frac{p_1^*}{p_2^*} < \frac{a_{11}}{a_{21}}$. Consider the case where $\frac{p_1^*}{p_2^*} = 0$. Then, it must be that $p_1^* = 0$. But then, $D(p_1^*, p_2^*)$ is not determined as $x_{1j}(p_1^*, p_2^*)$ is not determined, $i = 1, 2$, $j = 1, 2$, a contradiction. Consider the case where $\frac{p_1^*}{p_2^*} > 0$. Then, we have that

$$D(p_1^*, p_2^*) = \frac{p_2^*}{p_1^*} \neq 0 = S(p_1^*, p_2^*),$$

a contradiction. Hence, it must be that $\frac{p_1^*}{p_2^*} \geq \frac{a_{11}}{a_{21}}$. ■

We now state and prove a crucial proposition which fully characterizes the international equilibrium relative price of good 1 with respect to good 2 in terms of the total labor force and the production coefficients in both contries.

Proposition 9. *Under Assumptions 1, 2, 3, 4, and 5,* $\frac{p_1^*}{p_2^*} = \frac{a_{11}}{a_{21}}$ *if and only if* $\frac{a_{21}}{a_{22}} \leq \frac{\bar{L}_1}{\bar{L}_2}$; $\frac{a_{11}}{a_{21}} < \frac{p_1^*}{p_2^*} < \frac{a_{12}}{a_{22}}$ *if and only if* $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$; $\frac{p_1^*}{p_2^*} = \frac{a_{12}}{a_{22}}$ *if and only if* $\frac{\bar{L}_1}{\bar{L}_2} \leq \frac{a_{11}}{a_{12}}$.

Proof. Suppose that $\frac{p_1^*}{p_2^*} = \frac{a_{11}}{a_{21}}$. Then, we have that

$$D(p_1^*, p_2^*) = \frac{p_2^*}{p_1^*} = \frac{a_{21}}{a_{11}} = \frac{y_{11}^* + 0}{\frac{\bar{L}_1}{a_{21}} - \frac{a_{11}y_{11}}{a_{21}} + \frac{\bar{L}_2}{a_{22}}} = S(p_1^*, p_2^*).$$

But then, we have that

$$y_{11}^* = \frac{1}{2}\frac{\bar{L}_1}{a_{11}} + \frac{1}{2}\frac{a_{21}}{a_{11}}\frac{\bar{L}_2}{a_{22}}.$$

It must be that

$$y_{11}^* \leq \frac{\bar{L}_1}{a_{11}}.$$

Therefore, it must be that

$$\frac{a_{21}}{a_{22}} \leq \frac{\bar{L}_1}{\bar{L}_2}.$$

Suppose that $\frac{a_{11}}{a_{21}} < \frac{p_1^*}{p_2^*} < \frac{a_{12}}{a_{22}}$. Then, we have that

$$D(p_1^*, p_2^*) = \frac{p_2^*}{p_1^*} = \frac{\frac{\bar{L}_1}{a_{11}}}{\frac{\bar{L}_2}{a_{22}}} = S(p_1^*, p_2^*).$$

But then, we have that

$$\frac{a_{11}}{a_{21}} < \frac{\frac{\bar{L}_2}{a_{22}}}{\frac{\bar{L}_1}{a_{11}}} < \frac{a_{12}}{a_{22}}.$$

Therefore, it must be that

$$\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}.$$

Suppose that $\frac{p_1^*}{p_2^*} = \frac{a_{12}}{a_{22}}$. Then, we have that

$$D(p_1^*, p_2^*) = \frac{p_2^*}{p_1^*} = \frac{a_{22}}{a_{12}} = \frac{\frac{\bar{L}_1}{a_{21}} + y_{12}^*}{0 + \frac{\bar{L}_2}{a_{22}} - \frac{a_{12}y_{12}^*}{a_{22}}} = S(p_1^*, p_2^*).$$

But then, we have that

$$y_{12}^* = \frac{1}{2}\frac{\bar{L}_2}{a_{12}} - \frac{\bar{L}_1}{a_{11}}.$$

It must be that

$$y_{12}^* \geq 0.$$

Therefore, it must be that

$$\frac{\bar{L}_1}{\bar{L}_2} \leq \frac{a_{11}}{a_{12}}.$$

Suppose that $\frac{a_{21}}{a_{22}} \leq \frac{\bar{L}_1}{\bar{L}_2}$. Moreover, suppose that $\frac{a_{11}}{a_{21}} < \frac{p_1^*}{p_2^*} < \frac{a_{12}}{a_{22}}$. Then, it must be that $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, by the previous argument, a contradiction. Suppose now that $\frac{p_1^*}{p_2^*} = \frac{a_{11}}{a_{21}}$. Then it must be that $\frac{\bar{L}_1}{\bar{L}_2} \leq \frac{a_{11}}{a_{12}}$, by the previous argument, a contradiction. Therefore, it must be that $\frac{p_1^*}{p_2^*} = \frac{a_{11}}{a_{21}}$. Suppose that $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$. Moreover, suppose that $\frac{p_1^*}{p_2^*} = \frac{a_{11}}{a_{21}}$. Then, it must be that $\frac{a_{21}}{a_{22}} \leq \frac{\bar{L}_1}{\bar{L}_2}$, by the previous argument, a contradiction. Suppose now that $\frac{p_1^*}{p_2^*} = \frac{a_{12}}{a_{22}}$. Then, it must be that $\frac{\bar{L}_1}{\bar{L}_2} \leq \frac{a_{11}}{a_{12}}$, by the previous argument, a contradiction. Therefore, it must be that $\frac{a_{11}}{a_{21}} < \frac{p_1^*}{p_2^*} < \frac{a_{12}}{a_{22}}$. Suppose that $\frac{\bar{L}_1}{\bar{L}_2} \leq \frac{a_{11}}{a_{12}}$. Moreover, suppose that $\frac{p_1^*}{p_2^*} = \frac{a_{11}}{a_{21}}$ Then, it must be that $\frac{a_{21}}{a_{22}} \leq \frac{\bar{L}_1}{\bar{L}_2}$, by the previous argument, a contradiction. Suppose now that $\frac{a_{11}}{a_{21}} < \frac{p_1^*}{p_2^*} < \frac{a_{12}}{a_{22}}$. Then, it must be that $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, by the previous argument, a contradiction. Therefore, it must be that $\frac{p_1^*}{p_2^*} = \frac{a_{12}}{a_{22}}$. Hence, we have that $\frac{p_1^*}{p_2^*} = \frac{a_{11}}{a_{21}}$ if and only if $\frac{a_{21}}{a_{22}} \leq \frac{\bar{L}_1}{\bar{L}_2}$; $\frac{a_{11}}{a_{21}} < \frac{p_1^*}{p_2^*} < \frac{a_{12}}{a_{22}}$ if and only if $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$; $\frac{p_1^*}{p_2^*} = \frac{a_{12}}{a_{22}}$ if and only if $\frac{\bar{L}_1}{\bar{L}_2} \leq \frac{a_{11}}{a_{12}}$. ■

The next proposition, which easily follows from the previous arguments, exhibits the international equilibrium configuration when each country produces the good for which it has a comparative advantage.

Proposition 10. *Under Assumptions 1, 2, 3, 4, and 5, if* $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, *then* $\frac{p_1^*}{p_2^*} = \frac{\bar{L}_2/a_{22}}{\bar{L}_1/a_{11}}$, $x_{11}^* = \frac{\bar{L}_1}{2a_{11}}$, $x_{21}^* = \frac{\bar{L}_2}{2a_{22}}$, $x_{12}^* = \frac{\bar{L}_1}{2a_{11}}$, $x_{22}^* = \frac{\bar{L}_2}{2a_{22}}$.

The following proposition compares the autarky equilibrium and the international equilibrium when each country produces the good for which it has a comparative advantage, and shows that there are gains from trade for both countries.

Proposition 11. *Under Assumptions 1, 2, 3, 4, and 5, if* $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, *then* $u_j(x_{1j}^*, x_{2j}^*) > u_j(\tilde{x}_{1j}, \tilde{x}_{2j})$, $j = 1, 2$.

Proof. Suppose that $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$. Then, we have that

$$u_1(x_{11}^*, x_{21}^*) = \frac{\bar{L}_1 \bar{L}_2}{4a_{11}a_{22}} > \frac{\bar{L}_1^2}{4a_{11}a_{21}} = u_1(\tilde{x}_{11}, \tilde{x}_{21}),$$

as $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, and

$$u_2(x^*_{12}, x^*_{22}) = \frac{\bar{L}_1 \bar{L}_2}{4a_{11}a_{22}} > \frac{\bar{L}_1^2}{4a_{12}a_{22}} = u_2(\tilde{x}_{12}, \tilde{x}_{22}),$$

as $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2}$. ■

3 A Ricardo's fallacy?

This section is devoted to a discussion of the case where one country can produce only one good whereas the other produces both goods: it was proposed by Patnaik (2005) to affirm the existence of the Ricardo fallacy.

In order to incorporate this case in our version of Ricardo's model with Cobb-Douglas preferences defined in Section 2, we modify Assumption 2 as follows.

Assumption 2′. $0 < a_{i1} < +\infty$, $i = 1, 2$, $a_{12} = +\infty$, $0 < a_{22} < +\infty$.

The following proposition provides the computation of the autarky equilibrium for the modified version of the Ricardo model.

Proposition 12. *Under Assumptions 1, 2′, 3, and 4, at the autarky equilibrium, the quantities demanded and produced are* $\tilde{x}_{11} = \frac{\bar{L}_1}{2a_{11}} = \tilde{y}_{11}$, $\tilde{x}_{21} = \frac{\bar{L}_1}{2a_{21}} = \tilde{y}_{21}$, *and,* $\tilde{x}_{12} = 0 = \tilde{y}_{12}$, $x_{22} = \frac{\bar{L}_2}{2a_{22}} = \tilde{y}_{22}$.

The following proposition holds by the same argument used in the proof of Proposition 9, considering that $a_{12} = +\infty$, by Assumption 2′.

Proposition 13. *Under Assumptions 1, 2′, 3, and 4,* $\frac{p_1^*}{p_2^*} = \frac{a_{11}}{a_{21}}$ *if and only if* $\frac{a_{21}}{a_{22}} \leq \frac{\bar{L}_1}{\bar{L}_2}$; $\frac{a_{11}}{a_{21}} < \frac{p_1^*}{p_2^*}$ *if and only if* $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$.

The next proposition, which easily follows from the previous arguments, exhibits the international equilibrium configuration when country 1 specializes in the production of good 1.

Proposition 14. *Under Assumptions 1, 2′, 3, and 4, if* $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, *then* $\frac{p_1^*}{p_2^*} = \frac{\bar{L}_2/a_{22}}{\bar{L}_1/a_{11}}$, $x^*_{11} = \frac{\bar{L}_1}{2a_{11}}$, $x^*_{21} = \frac{\bar{L}_2}{2a_{22}}$, $x^*_{12} = \frac{\bar{L}_1}{2a_{11}}$, $x^*_{22} = \frac{\bar{L}_2}{2a_{22}}$.

The following proposition compares the autarky equilibrium and the international equilibrium when country 1 specializes in the production of good 1, and shows that there still are gains from trade for both countries.

Proposition 15. *Under Assumptions 1, 2′, 3, and 4, if* $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$*, then* $u_j(x^*_{1j}, x^*_{2j}) > u_j(\tilde{x}_{1j}, \tilde{x}_{2j})$*,* $j = 1, 2$*.*

Proof. Suppose that $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$. Then, we have that

$$u_1(x^*_{11}, x^*_{21}) = \frac{\bar{L}_1\bar{L}_2}{4a_{11}a_{22}} > \frac{\bar{L}_1^2}{4a_{11}a_{21}} = u_1(\tilde{x}_{11}, \tilde{x}_{21}),$$

as $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, and

$$u_2(x^*_{12}, x^*_{22}) = \frac{\bar{L}_1\bar{L}_2}{4a_{11}a_{22}} > 0 = u_2(\tilde{x}_{12}, \tilde{x}_{22}),$$

as $\tilde{x}_{12} = 0$. ■

Proposition 15 contradicts the asserted claim of Patnaik (2005) that there is a Ricardo's fallacy when one of the countries produces only one good.

Nonetheless, the analysis developed here under the assumption of Cobb-Douglas preferences confirms Patnaiks statement that, when both countries produce both goods, the total output of the two goods increases vector-wise after international trade with respect to autarky. This can easily seen by considering that, if $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, we have that

$$x^*_{11} + x^*_{12} = \frac{\bar{L}_1}{a_{11}} > \frac{a_{12}\bar{L}_1 + a_{11}\bar{L}_2}{2a_{11}a_{12}} = \tilde{x}_{11} + \tilde{x}_{21},$$

as $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2}$, and

$$x^*_{21} + x^*_{22} = \frac{\bar{L}_2}{a_{22}} > \frac{a_{22}\bar{L}_1 + a_{21}\bar{L}_2}{2a_{11}a_{12}} = \tilde{x}_{11} + \tilde{x}_{21},$$

as $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$.

Patnaik (2005) supported her statement with the argument that the bundle of goods available to each country must be vector-wise larger with trade than without trade. Indeed, if $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, we have that

$$x^*_{11} = \frac{\bar{L}_1}{2a_{11}} = \tilde{x}_{11},$$

$$x^*_{21} = \frac{\bar{L}_2}{2a_{22}} > \frac{\bar{L}_1}{2a_{21}} = \tilde{x}_{21},$$

as $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, and

$$x_{12}^* = \frac{\bar{L}_1}{2a_{11}} > \frac{\bar{L}_1}{2a_{11}} = \tilde{x}_{12},$$

$$x_{22}^* = \frac{\bar{L}_2}{2a_{11}} = \tilde{x}_{22},$$

as $\frac{a_{11}}{a_{12}} < \frac{\bar{L}_1}{\bar{L}_2}$.

In this regard, it must be stressed that Patnaik (2005) is implicitly assuming that preferences in each country are strongly monotone. Only under this assumption she can conclude that the vector-wise increase in the bundle of goods available to each country after international trade is a sufficient condition for gains from trade. Clearly, her argument holds in the context of our model, since our Mill-Graham preferences, represented by a Cobb-Douglas utility function with equal expenditure shares, are strongly monotone except on the boundary of R_+^2.

Our analysis confirms also Patanik's statement that, when country 2 does not produce good 1, the total output of the two goods does not increase vector-wise after international trade with respect to autarky. Indeed, if $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, we have that

$$x_{11}^* + x_{12}^* = \frac{\bar{L}_1}{a_{11}} > \frac{\bar{L}_1}{2a_{11}} = \tilde{x}_{11} + \tilde{x}_{21},$$

and

$$x_{21}^* + x_{22}^* = \frac{\bar{L}_2}{a_{22}} < \frac{\bar{L}_2}{a_{21}} + \frac{\bar{L}_2}{a_{22}} = \tilde{x}_{11} + \tilde{x}_{21}.$$

Patnaik (2005) supported her statement with the argument that the bundle of goods available to country 2 cannot be vector-wise larger with trade than without trade. Indeed, if $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, we have that

$$x_{11}^* = \frac{\bar{L}_1}{2a_{11}} = \tilde{x}_{11},$$

$$x_{21}^* = \frac{\bar{L}_2}{2a_{22}} > \frac{\bar{L}_1}{2a_{21}} = \tilde{x}_{21},$$

as $\frac{\bar{L}_1}{\bar{L}_2} < \frac{a_{21}}{a_{22}}$, and

$$x_{12}^* = \frac{\bar{L}_1}{2a_{11}} > 0 = \tilde{x}_{12},$$

$$x_{22}^* = \frac{\bar{L}_2}{2a_{11}} < \frac{\bar{L}_2}{2a_{22}} = \tilde{x}_{22}.$$

The fact that the bundle of goods available to country 2 cannot be vector-wise larger with trade than without trade led Patnaik (2005) to conclude that, when country 2 does not produce good 1, there cannot be gains from trade, thereby exhibiting a material fallacy in Ricardo's theory of comparative advantage. Nevertheless, this conclusion is itself based on the fallacy that, with strongly monotone preferences in each country, a vector-wise increase in the bundle of goods available to each country after international trader is a necessary condition for gains from trade: this condition is indeed sufficient but not necessary for gains from trade. This is why our Proposition 15 shows that, with strongly monotone Mill-Graham-Cobb-Douglas preferences in both countries, there are gains from trade even if the bundle of goods available to a country is not vector-wise larger with trade than without trade.

4 Conclusion

In this paper, we have considered a crucial aspect of the theory of imperialism developed in a complete way by Patnaik and Patnaik (2017), namely, the failure of the Ricardian theory of comparative advantage, already analyzed by Patnaik (2005), when there is a material asymmetry due to the fact that some goods are produced almost exclusively in the global south.

We have dealt with this issue through a formal analysis of the Ricardo model of international trade with two goods, two countries, and with Mill-Graham preferences, represented by a Cobb-Douglas utility function with equal expenditure shares, and we have shown that there are gains from international trade despite of the material asymmetry, thereby showing that there is no Ricardo's fallacy either in this case.

This does not imply that there is no other type of exploitation that the "global north" can exercise on the "global south" via international trade. For instance, as pointed out by Heinisch (2006), some Sub-Saharan countries – Benin, Burkina Faso, and Mali – successfully challenged US cotton subsidies, which squeezed their potential gains from trade, exploiting the liberal economic principles inspired by the Ricardo theory of comparative advantage embodied by international institutions as the World Trade Organization.

We leave for further research an analysis of the other implications of our findings for the whole theory of imperialism developed by Patanik and Patnaik (2017).

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