



# **RASCH ANALYSIS OF THE HOPE-SSQ QUESTIONNAIRE**

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## Abstract

The data arising from the "HOPE –WG1 - SSQ - Questionnaire", gathered on almost 1500 students in 27 sites around Europe, has been analyzed using Rasch models, in order to extract and measure factors inspiring to study physics. In particular, using a Rating Scale Model (Wright & Masters, 1982) and Principal Components Analysis (PCA) of standardized residuals, we identified and measured two main latent traits: FACTOR A, characterized by curiosity for the world and interest in its comprehension, and FACTOR B, characterized by the wish to get an interesting job, enhancing employment prospects, stimulus from visiting museums, laboratories, seeing things on the TV or internet, learning interesting things at schools, receiving encouragement from parent and friend and information during visit from universities staff. These factors, interacting with other personal characteristics such as sex, level of knowledge of physics and so on, may influence performance, decisions, goals and preferences. We applied multilevel logistic regression models with SIMEX correction (Lederer & Küchenhoff, 2006), using the estimated factors as explanatory variables: the results show that these are significant and relevant in explaining the decision to study physics, in association with the level of knowledge of physics and the wish to become a physics teacher. Some possible guidelines for stimulating the decision to study physics arises from this analysis.

**Keywords:** Hope-SSQ questionnaire, Rasch models, Latent traits, Multilevel logistic regression, SIMEX correction

## Aim of the study

SSQ-HOPE questionnaire is one of the actions of the EU HOPE-Project (Horizons in physics education), a cooperation project of 71 European partners. The HOPE-project is striving to find ways to inspire young people to study physics (Working Group 1 - WG1). SSQ is part of it in concentrating on the transition school-university with a focus on the factors motivating secondary school students, which are talented in physics to study physics. For more information regarding the survey and some preliminary results see Michelini et al. (2016). Personality and cognitive traits have been recognized important factors for success and decision making in many aspects of life, such as investment in education and human capital (Heckman et al., 2006; Battauz, 2006; Gori, 2004). Empirical evidences of such relation are relatively recent, and the recognition of the importance of traits other intelligence leads to a growing interest to identify which traits are important for which outcome. Economists can profitably leverage research from psychology on measurement, prediction and malleability of personality traits (Borghans et al., 2008). As it will be clear from the subsequent analysis, the HOPE-SSQ questionnaire allows measuring personality traits that, among other factors, may influence the decision to study physics, and other relevant aspects such as cognitive ability in physics and the desire to become a physics teacher. The methodology used to this end is that of Rasch models (Rasch, 1960) which have the property of producing interval scale, objective measures of the traits of persons, from ordinal observations (the answers to the questionnaire). "Objective measurement is the repetition of a unit amount that maintains its size, within an allowable range of error, no matter which instrument, intended to measure the variable of interest, is used and no matter who or what relevant person or thing is measured" (<http://www.rasch.org/define.htm>). The Rasch models satisfy such definition thanks to their Specific Objectivity property according to which "comparisons between individuals become independent of which particular instruments -- tests or items or other stimuli -- have been used. Symmetrically, it ought to be possible to compare stimuli belonging to the same class -- measuring the same thing -- independent of which particular individuals, within a class considered, were instrumental for comparison." (Rasch, 1977). Other methods such as Classical

Test Theory, Factor Analysis and IRT models do not satisfy the objectivity criteria and will not be considered here.

## Data and methods

The Hope SSQ questionnaire has been prepared to collect inspiring Factors to study physics by secondary school students oriented to a physics degree. Data collection took place in a series of special events for talent students in physics. For the present analysis, we considered Part A and Part C of the questionnaire.

Part A: Please give a score from 1 (not important at all) to 5 (very important) for each of the following aspects.	Part C: Please give a score from 1 (not important at all) to 5 (very important) for each of the following events or reasons, which may inspire you to study physics.
A1. I think that the physics lessons are important for the culture of the citizen	C1. A wish to acquire a deep understanding of the universe
A2. During physics lessons I learn useful things	C2. A wish to enhance employment prospects
A3. I enjoy physics lessons	C3. Encouragement from friends/classmates
A4. I have some insight what a physicist does in his/her work in physics research	C4. A physics teacher in school
A5. I have some insight into the goals of physics research	C5. Seeing TV documentaries on physics topics
A 6. I have some insight into the meaning of physics research for daily life	C6. Reading books or magazines
A7. I have some insight what a physicist does if working outside university	C7. Being inspired by a scientist in your family
A8. I like to be engaged in physics in my free time	C8. A wish to understand the world around you
A9. I got informed about the latest research in physics	C9. Visits to museums or special exhibitions
Legend:  FACTOR A  FACTOR B	C10. Visits to scientific laboratories, e.g. universities, CERN, ...
	C11. Visits from university staff or students to your School
	C12. Seeing things on the internet e.g. websites, YouTube
	C13. Wanting to understand how things work
	C14. A wish to learn advanced physics (e.g. quantum mechanics)
	C15. Making and/or using a physics-based device e.g. a telescope
	C16. A wish to get an interesting job
	C17. A wish to become a physics researcher
	C18. A wish to become a physics teacher
	C19. Physics is the school subject I do best at now
	C20. Encouragement from parents or family

We started, analyzing the two groups of items as a single group, searching for a common latent trait. We excluded a priori the item C19. *Physics is the school subject I do best at now*, because this question represents the degree of cognitive ability in Physics and it would be worthwhile to analyze this as a separated dimension. Using a Rasch Rating Scale model, we investigated the eventual multidimensionality of the latent trait, by Principal Components Analysis (PCA) of standardized residuals (Linacre, 2009). The dimensions found by Rasch PCA are then analyzed separately and interpreted on the base of their content. In particular, two dimensions of interest were found: FACTOR A and FACTOR B. The measures obtained for such dimensions will then be analyzed on the light of the main characteristics of the student such as GENDER, C22. *At what age did you first become very interested in physics?* and C19. *Physics is the school subject I do best at now*. We will then concentrate our attention to explain an event of particular interest: C24. *Have you decided to apply to study physics at university?* (YES/NO). In particular, we will apply multilevel logistic regression models to explain the probability of such event as function of the main explanatory variables described above, using the site of interview as grouping variable, to account for the multilevel structure of the data. Being FACTOR A and FACTOR B estimated by Rasch model, and therefore affected by error, we adopted a SIMEX approach to correct for bias in estimated coefficients.

## Rasch analysis of the data

Being the data expressed on a Likert scale with 5 levels, we applied a Rasch Rating Scale Model (Andrich, 1978; Wright & Masters, 1982)

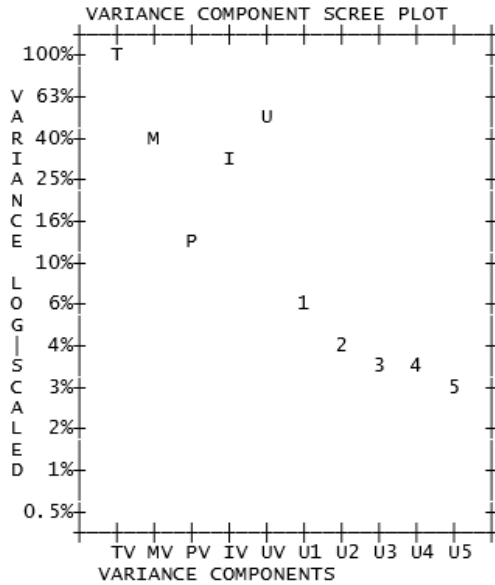
$$(1) P(X_{ni} = x) = \frac{\exp \sum_{k=0}^x [\beta_n - (\delta_i + \tau_k)]}{\sum_{j=0}^m \exp \sum_{k=0}^j [\beta_n - (\delta_i + \tau_k)]}, \quad x = 0, 1, 2, \dots, m,$$

where  $P(X_{ni} = x)$  is the probability that the individual  $n$  respond  $x$  to the question  $i$ ;  $\beta_n$  is the so called "ability" of the individual  $n$  (i.e. in this case the level of the latent trait that we want to measure),  $\delta_i$  is the "difficulty" of the question (item)  $i$  (in practice how rare is to find an high score on this item),  $\tau_k$  is the "difficulty" to reach level  $x = k$ , common to all items;  $m$  is the maximum score. From a first run of Winsteps (Linacre, 2016), one of the most famous software for Rasch Analysis (Bond & Fox, 2007), we obtained the following table 23.0 and we found that the largest eigenvalue of PCA was 3.52, evidencing possible multidimensionality and/or violation of local independence hypothesis. However the maximum correlation for the standardized residuals was around 0.40: not being very high ( $>0.70$ ) we concluded that the local independence hypothesis was not violated. In a dataset, fitting the Rasch model, we have a variability that is due to the model and a residual variability due to randomness. Rasch "PCA of residuals" looks for patterns in the part of the data due to randomness. This eventual pattern is the "unexpected" part of the data that may be due, among other reasons (Smith, 2002), to the presence of multiple dimensions in the data. In the Rasch PCA of residuals, we are looking for groups of items sharing the same patterns of unexpectedness. In particular, the matrix of item correlations based on residuals is decomposed to identify possible "contrasts" (the principal components) that may be affecting response patterns. Usually the contrast needs to have the strength (eigenvalue) of at least two items to be above the noise level: in our case, the first contrast (i.e. the contrast with the highest eigenvalue) has a strength of 3.52 (see Tables 23.0-1), so almost four items.

Table of STANDARDIZED RESIDUAL variance in Eigenvalue units = ITEM information units

	Eigenvalue	Observed	Expected
Total raw variance in observations =	51.8276	100.0%	100.0%
Raw variance explained by measures =	23.8276	46.0%	46.6%
Raw variance explained by persons =	7.4137	14.3%	14.5%
Raw variance explained by items =	16.4139	31.7%	32.1%
Raw unexplained variance (total) =	28.0000	54.0%	53.4%
Unexplned variance in 1st contrast =	3.5164	6.8%	12.6%
Unexplned variance in 2nd contrast =	2.1904	4.2%	7.8%
Unexplned variance in 3rd contrast =	1.8532	3.6%	6.6%
Unexplned variance in 4th contrast =	1.6449	3.2%	5.9%
Unexplned variance in 5th contrast =	1.5637	3.0%	5.6%

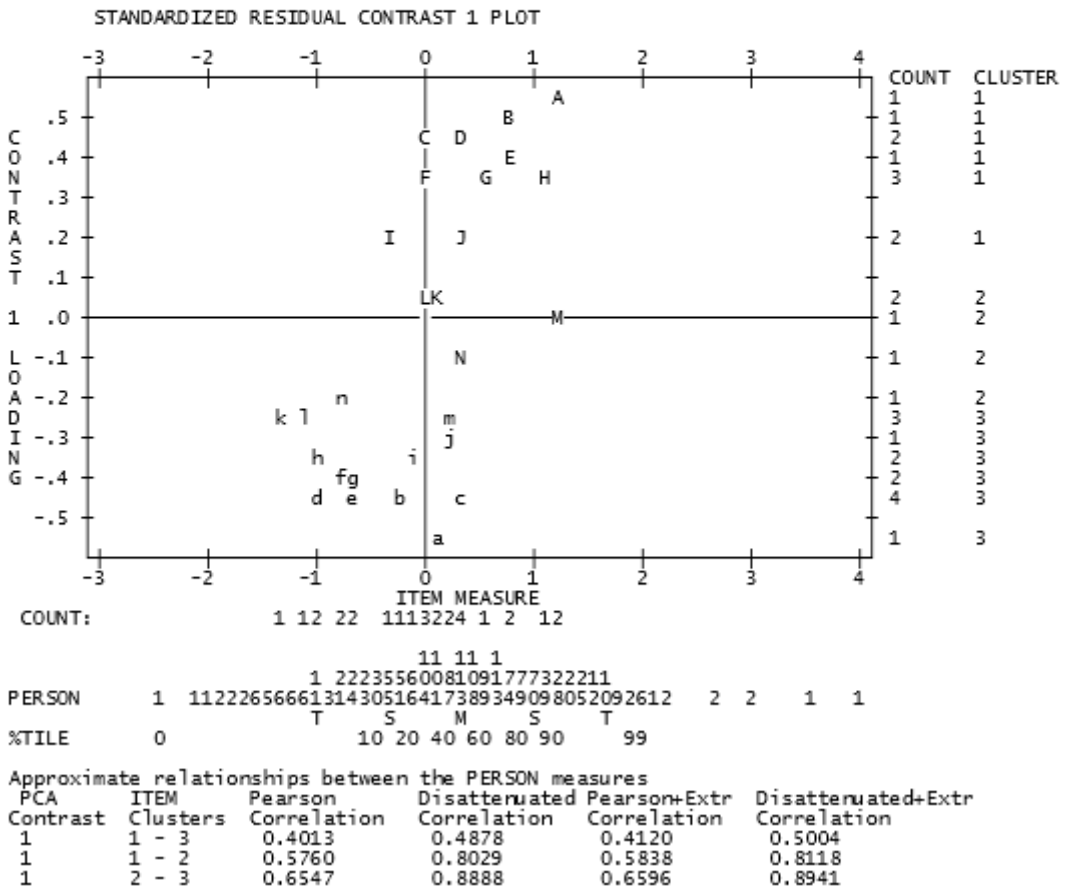
STANDARDIZED RESIDUAL VARIANCE SCREE PLOT



As we may see from Table 23.1 of Winsteps, the disattenuated correlation between the person measures obtained using the items of the opposite clusters 1, corresponding to the highest loading, and 3, corresponding to the lowest loading, is only 0.49 (a unique dimension would suggest a correlation of 1). In order to confirm the presence of separated dimensions we contrasted the content of the items at the top, A, B, C, and at the bottom, a, b, c, of the contrast plot in Table 23.1. If those items (see Winsteps Table 23.2) are different enough to be considered different dimensions (similar to "height" and "weight"), then we can split the items into separate analyses.

Table of STANDARDIZED RESIDUAL variance in Eigenvalue units = ITEM information units

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Raw variance explained by persons =	7.4137	14.3%	14.5%
Raw Variance explained by items =	16.4139	31.7%	32.1%
Raw unexplained variance (total) =	28.0000	54.0%	53.4%
Unexplned variance in 1st contrast =	3.5164	6.8%	12.6%



Indeed, we may see that the one at the bottom, such as A8. *I like to be engaged in physics in my free time*, A9. *I got informed about the latest research in physics*, C14. *A wish to learn advanced physics (e.g. quantum mechanics)*, can be denoted as FACTOR A, while that at the top such as C2. *A wish to enhance employment prospects*, C3. *Encouragement from friends/classmates*, C11. *Visits from university staff or students to your School*, are related to a dimension that we could call FACTOR B.

TABLE 23.2 HOPE SSQ questionnaire ZOU920WS.TXT May 24 2016 14:49  
 INPUT: 1430 PERSON 30 ITEM REPORTED: 1430 PERSON 28 ITEM 140 CATS WINSTEPS 3.92.0

CONTRAST 1 FROM PRINCIPAL COMPONENT ANALYSIS  
 STANDARDIZED RESIDUAL LOADINGS FOR ITEM (SORTED BY LOADING)

CON-TRAST	LOADING	INFIT OUTFIT			ENTRY		LOADING	INFIT OUTFIT			ENTRY	
		MEASURE	MNSQ	MNSQ	NUMBER	ITE		MEASURE	MNSQ	MNSQ	NUMBER	ITE
1	.55	1.19	1.15	1.25	A	3 C3	-.54	.14	.82	.81	a	28 A8
1	.52	.81	1.03	1.04	B	11 C11	-.45	-.27	.86	.87	b	14 C14
1	.47	.02	1.34	1.40	C	2 C2	-.44	.37	.90	.89	c	29 A9
1	.44	.37	1.17	1.24	D	4 C4	-.43	-1.05	.91	.90	d	23 A3
1	.39	.80	1.06	1.16	E	20 C20	-.43	-.62	1.01	1.00	e	25 A5
1	.37	.03	.99	.99	F	10 C10	-.41	-.75	.96	.97	f	1 C1
1	.34	.52	1.02	1.03	G	9 C9	-.38	-.65	1.03	1.01	g	26 A6
1	.33	1.16	1.27	1.53	H	7 C7	-.36	-1.03	.92	.94	h	8 C8
1	.22	-.35	1.01	1.02	I	16 C16	-.34	-.12	.99	.98	i	24 A4
1	.20	.33	.96	.96	J	5 C5	-.28	.26	.84	.84	j	17 C17
1	.05	.15	.98	.98	K	12 C12	-.27	-1.29	.85	.82	k	13 C13
1	.05	.02	.95	.97	L	15 C15	-.26	-1.07	.98	.99	l	22 A2
							-.25	.22	1.02	1.02	m	27 A7
							-.18	-.80	1.08	1.08	n	21 A1
							-.08	.35	.92	.91	N	6 C6
							-.01	1.27	1.16	1.15	M	18 C18

Therefore, we decided to split the analysis in two using the items of cluster 1 and cluster 3, as suggested by Table 23.1, and assigning the item of cluster 2, to one of the two groups, on the base of the content and of the fit indices. After several runs of Winsteps, excluding misfitting persons and items, we ended up with the following results.

### Main results for the FACTOR A scale

The FACTOR A scale was formed by 8 items shown in Table 13.1. The hardest items are A9. *I got informed about the latest research in physics* and C17. *A wish to become a physics researcher*. The easiest items, instead, are C13. *Wanting to understand how things work* and C8. *A wish to understand the world around you*. These latter are clearly the base for the former.

TABLE 13.1 HOPE SSQ questionnaire - Attitude ZOU403WS.TXT Jun 4 2016 14:51  
 INPUT: 1430 PERSON 30 ITEM REPORTED: 1340 PERSON 8 ITEM 5 CATS WINSTEPS 3.92.0

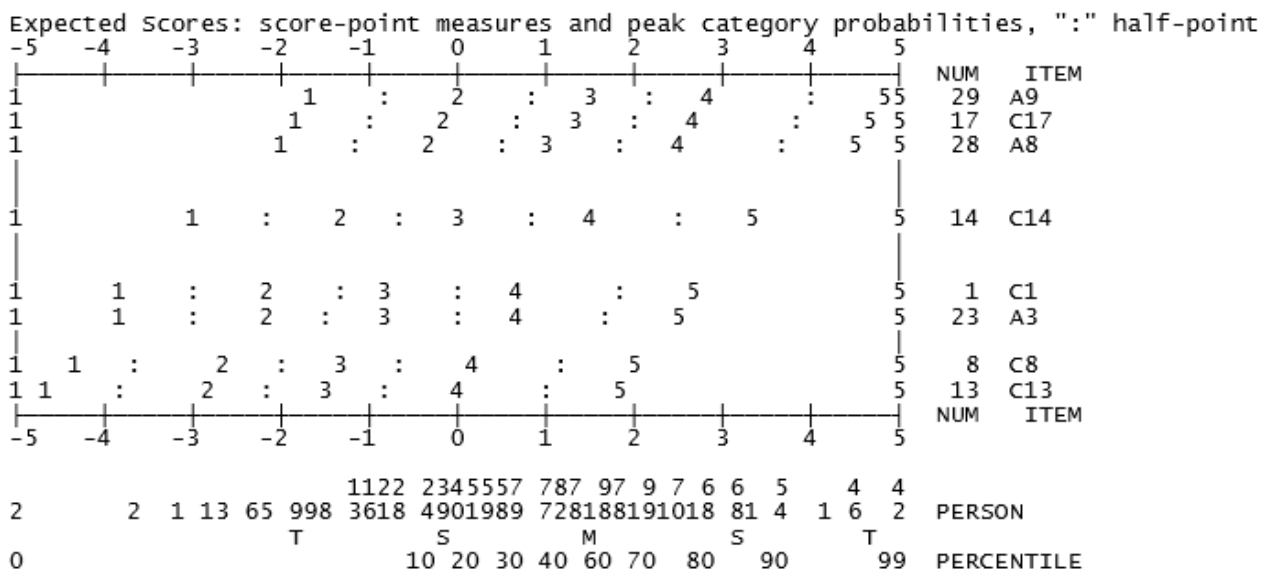
PERSON: REAL SEP.: 2.20 REL.: .83 ... ITEM: REAL SEP.: 27.16 REL.: 1.00

ITEM STATISTICS: MEASURE ORDER

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT		OUTFIT		PTMEASUR-AL CORR. EXP.		EXACT OBS%	MATCH EXP%	ITEM
					MNSQ	ZSTD	MNSQ	ZSTD					
29	3981	1339	1.50	.04	1.07	1.9	1.10	2.5	.71	.76	45.1	46.2	A9
17	4115	1333	1.32	.04	1.13	3.2	1.09	2.2	.76	.76	44.4	46.2	C17
28	4274	1339	1.14	.04	.85	-4.1	.83	-4.7	.79	.75	51.9	47.1	A8
14	5062	1339	.12	.04	1.06	1.6	.98	-.5	.74	.71	50.0	50.7	C14
1	5577	1337	-.69	.04	1.01	.2	.92	-1.5	.68	.66	57.1	57.2	C1
23	5593	1337	-.71	.04	1.06	1.4	1.28	4.9	.62	.66	59.7	57.3	A3
8	5874	1338	-1.25	.05	.90	-2.3	.85	-2.4	.64	.62	63.6	62.6	C8
13	5966	1338	-1.44	.05	.83	-3.8	.81	-2.9	.63	.61	67.3	65.1	C13
MEAN	5055.3	1337.5	.00	.04	.99	-.2	.98	-.3			54.9	54.0	
P. SD	767.3	1.9	1.11	.00	.11	2.6	.15	3.0			7.9	7.1	

The construct-key map of Table 2.2 shows us that, in order to assign grade 5 to the A9 item, you need to have an FACTOR A (measure) of at least 4: a level above the 90<sup>th</sup> percentile. A FACTOR A equal to 2 (the 70<sup>th</sup> percentile) implies that a student answered with four or five to all questions except A9 (answered with 3). The mean level for person is around 1.50, meaning that this test is relatively easy: some effort could be spent in finding some more difficult items if we wish to build a more complete scale for FACTOR A.

TABLE 2.2 HOPE SSQ questionnaire - Attitude ZOU403WS.TXT Jun 4 2016 14:51  
 INPUT: 1430 PERSON 30 ITEM REPORTED: 1340 PERSON 8 ITEM 5 CATS WINSTEPS 3.92.0



The psychometric properties of this scale are quite good. First, we observe that from the analysis of Rasch PCA residuals the highest eigenvalue is (only) 1.80: being less than 2 this means that the construct is unidimensional. All fit indices are very close to 1 and in the range (0.60-1.40) suggested (Bond & Fox, 2007) for rating scale analysis. The reliability index for the items was 1.00, while that for persons was 0.86 with a Cronbach-alpha of 0.87. The category structure of the Rating Scale Model used here are reported in Table 3.2 and as we see the Andrich threshold (Linacre, 2001) are well ordered, with good fit indices. Interesting to note that when we added the item C18. *A wish to become a physics teacher* it did not fit the model (INFIT an OUTFIT around 2): this means that FACTOR A (related to research) and teaching are different dimensions.

TABLE 3.2 HOPE SSQ questionnaire - Attitude ZOU403WS.TXT Jun 4 2016 14:51  
 INPUT: 1430 PERSON 30 ITEM REPORTED: 1340 PERSON 8 ITEM 5 CATS WINSTEPS 3.92.0

SUMMARY OF CATEGORY STRUCTURE. Model="R"

CATEGORY LABEL	OBSERVED SCORE	OBSERVED COUNT	OBSERVED %	OBSVD AVRG	SAMPLE EXPECT	INFIT MNSQ	OUTFIT MNSQ	ANDRICH THRESHOLD	CATEGORY MEASURE
1	1	597	6	-1.66	-1.73	1.11	1.11	NONE	( -3.14)
2	2	1144	11	-.66	-.65	1.03	1.12	-1.84	-1.43
3	3	2109	20	.32	.36	.94	.90	-.76	-.06
4	4	3020	28	1.51	1.51	.99	.93	.56	1.41
5	5	3830	36	2.94	2.92	.98	.99	2.04	( 3.29)
MISSING		20	0	.99					

OBSERVED AVERAGE is mean of measures in category. It is not a parameter estimate.

In order to check the validity of the scale, we performed an analysis of the Differential Item Functioning (DIF) (Holland & Wainer, 1993), to see if the difficulty of the items vary among the locations of the survey. Differences in the difficulties of the items could in fact destroy the objectivity properties of the scale. Fig.1 shows the size of the item difficulties among the locations and Fig. 2 the t-test for the difference with respect to the average difficulty (in evidence the band interval -2.58 , +2.58). As we see, some locations differ quite strongly from each other. In order to understand if these differences are relevant to measure the FACTOR A of persons we selected the locations with the highest value of the t-tests and we looked at the correlation between person's measures obtained using the average difficulties and that obtained using difficulties estimated only with the data relative to the specific location.



Fig. 1

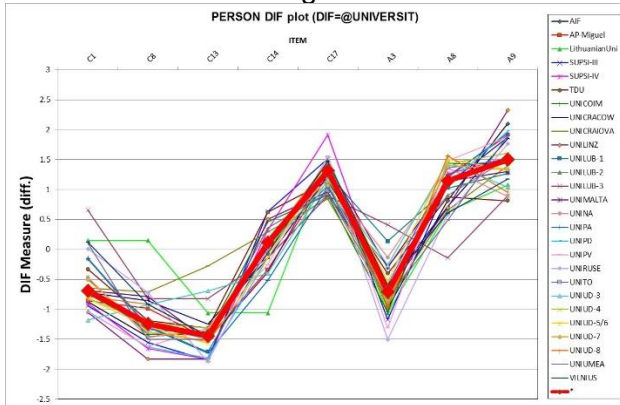
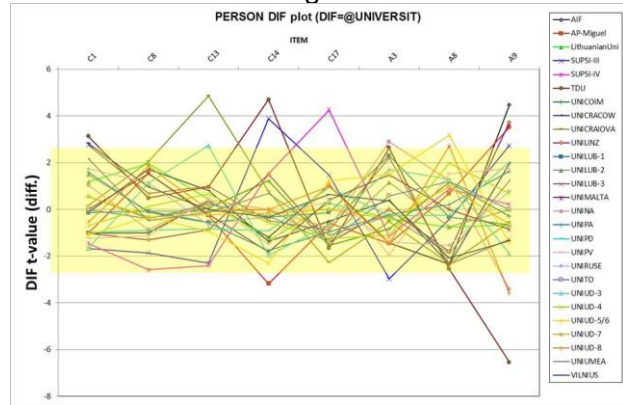


Fig. 2



For the locations with the largest and significant DIF: TUD, SUPSI-IV, UNICRAIOVA, the disattenuated correlation was 1, meaning that the consideration of the specific difficulties for the location does not change the measures of the persons.

**Main results for the FACTOR B scale**

14 items, shown in Table 13.1, formed the FACTOR B scale. The hardest item is C3. *Encouragement from friends/classmates*, followed by C20. *Encouragement from parents or family*, and C11. *Visits from university staff or students to your School*. The easiest items, instead, are A2. *During physics lessons, I learn useful things*, A1. *I think that the physics lessons are important for the culture of the citizen*, C16. *A wish to get an interesting job*. This means that in order the encouragement from friends, parents and institutions be important for FACTOR B a person must primarily feel that physics is useful for job and culture.

TABLE 13.1 HOPE 5SQ questionnaire - Motivation ZOU435WS.TXT Jun 9 2016 14: 9  
 INPUT: 1430 PERSON 30 ITEM REPORTED: 1341 PERSON 14 ITEM 5 CATS WINSTEPS 3.92.0

PERSON: REAL SEP.: 2.16 REL.: .82 ... ITEM: REAL SEP.: 19.77 REL.: 1.00

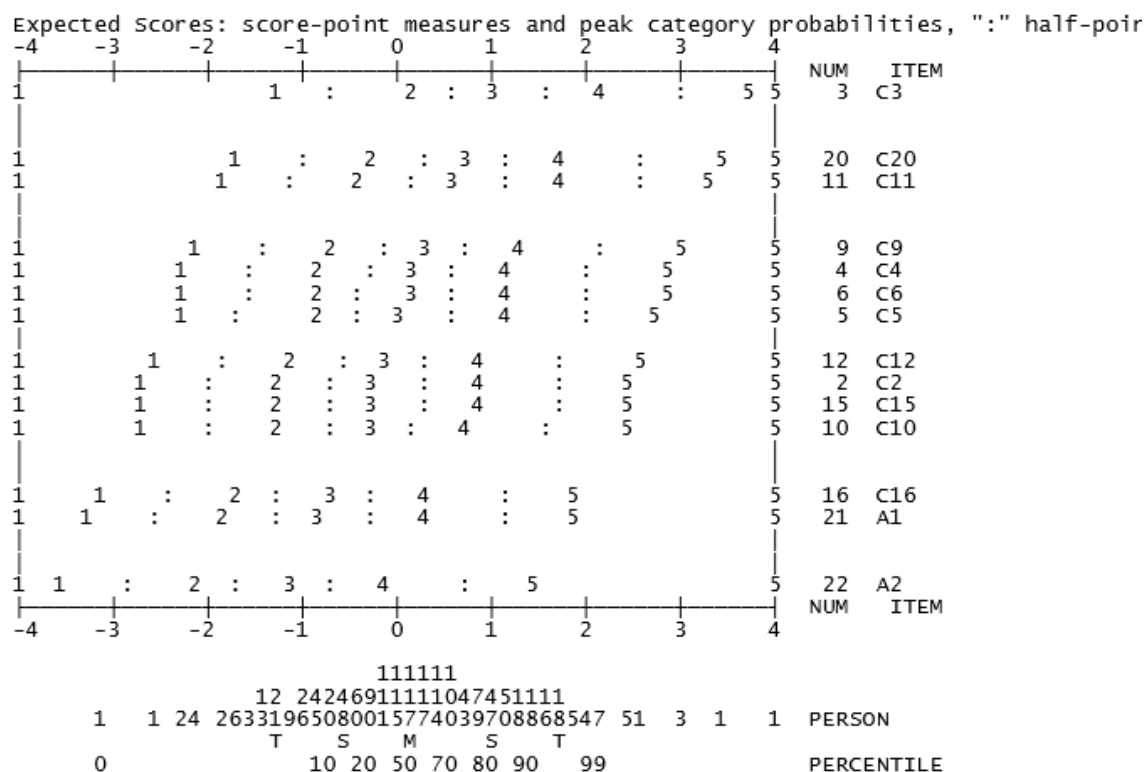
ITEM STATISTICS: MEASURE ORDER

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S. E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PTMEASUR-CORR.	AL-EXP.	EXACT OBS%	MATCH EXP%	ITEM
3	2932	1336	1.11	.03	.90	-2.9	.90	-2.7	.56	.55	43.8	39.1	C3
20	3341	1339	.77	.03	1.01	.4	1.01	.2	.60	.57	38.1	37.3	C20
11	3466	1335	.66	.03	.94	-1.8	.92	-2.2	.65	.57	36.2	37.0	C11
9	3886	1339	.34	.03	.86	-4.1	.88	-3.6	.59	.57	38.8	36.6	C9
4	4058	1337	.19	.03	1.12	3.3	1.12	3.3	.57	.57	35.5	37.1	C4
6	4106	1339	.16	.03	.93	-2.0	.95	-1.5	.57	.57	36.8	37.1	C6
5	4178	1340	.11	.03	.85	-4.6	.84	-4.8	.63	.57	41.4	37.4	C5
12	4435	1340	-.10	.03	1.01	.3	1.00	.0	.57	.57	35.3	38.3	C12
2	4536	1337	-.19	.03	1.16	4.2	1.17	4.5	.48	.56	35.5	38.9	C2
15	4553	1338	-.20	.03	1.07	1.8	1.07	1.9	.55	.56	38.4	38.9	C15
10	4602	1335	-.25	.03	1.02	.4	1.00	.0	.63	.56	35.7	39.2	C10
16	5125	1338	-.71	.03	1.14	3.6	1.11	2.7	.56	.53	38.0	42.2	C16
21	5159	1336	-.76	.03	1.05	1.4	1.11	2.6	.37	.53	41.7	42.4	A1
22	5516	1338	-1.13	.03	1.00	.1	1.02	.5	.42	.50	48.8	46.2	A2
MEAN	4278.1	1337.6	.00	.03	1.00	.0	1.01	.1			38.9	39.1	
P. SD	701.5	1.6	.60	.00	.10	2.7	.10	2.7			3.7	2.6	

The construct-key map of Table 2.2 shows us that, in order to assign grade 5 to the item C3, you need to have a FACTOR B of at least 3: a level above the 99<sup>th</sup> percentile. A FACTOR B equal to 1.5 (the 95<sup>th</sup> percentile) imply that a student answered with four or five to all questions, with the exception of item 3. The mean level for person is around zero, meaning that this test is well calibrated. A particular consideration deserve the item C11. *Visits from university staff or students to your School*,

which is one of the most difficult to endorse. Here one may think that this kind of instrument (visit from university staff) is not effective because a few students declare a high score to this question. But the structure of the scale suggests another story: in order to declare for example 4 or 5 to this question the FACTOR B of the student must be at level 2.5, as we may see from the construct-key map of Table 2.2. This means that in order such an action as C11 be effective it must meet students that are already motivated by the items of the scale easier to endorse, such as C9. *Visits to museums or special exhibitions*. On the other end, we may think that the hardness of C11 is the result of the rarity of the event. In any case, both interpretations do not suggest that the action C11 is ineffective.

TABLE 2.2 HOPE SSQ questionnaire - Motivation ZOU435WS.TXT Jun 9 2016 14: 9  
 INPUT: 1430 PERSON 30 ITEM REPORTED: 1341 PERSON 14 ITEM 5 CATS WINSTEPS 3.92.0



The psychometric properties of this scale are good. First, we observe that from the analysis of Rasch PCA residuals the highest eigenvalue is (only) 1.85: being less than 2 this means that the construct is unidimensional. All fit indices are very close to 1 and in the range (0.60-1.40). The reliability index for the items was 1.00, while that for persons was 0.85 with a Cronbach-alpha of 0.84. The category structure of the Rating Scale Model are reported in Table 3.2, and as we may see the Andrich threshold are well ordered, with good fit indices.

SUMMARY OF CATEGORY STRUCTURE. Model="R"

CATEGORY LABEL	OBSERVED SCORE	OBSERVED COUNT	OBSERVED %	OBSVD AVRGE	SAMPLE EXPECT	INFIT MNSQ	OUTFIT MNSQ	ANDRICH THRESHOLD	CATEGORY MEASURE	
1	1	2256	12	-.97	-.92	.95	.96	NONE	( -2.46)	1
2	2	3330	18	-.38	-.40	1.01	1.01	-1.05	-1.03	2
3	3	4774	25	.10	.08	1.01	1.03	-.52	-.06	3
4	4	5180	28	.57	.57	1.02	1.02	.24	.99	4
5	5	3187	17	1.10	1.12	1.03	1.02	1.33	( 2.63)	5
MISSING		47	0	.33						

OBSERVED AVERAGE is mean of measures in category. It is not a parameter estimate.

In order to check the validity of the scale, we performed an analysis of the Differential Item Functioning (DIF), to see if the difficulty of the items vary among the locations of the survey: Fig.3 shows the size of the item difficulties among the locations and Fig. 4 the t-test for the difference with respect to the average difficulty (in evidence the band interval -2.58 , +2.58). We see that LithuanianUni, that exhibits the major difference from Fig. 3, is not statistically significant from Fig. 4. Other locations exhibit a statistically different behavior. In order to understand if these differences are relevant to measure the FACTOR B of persons we selected the locations with the highest value of the t-tests and we looked at the correlation between person's measures obtained using the average difficulties and that obtained using difficulties estimated only with the data relative to the specific location.

Fig. 3

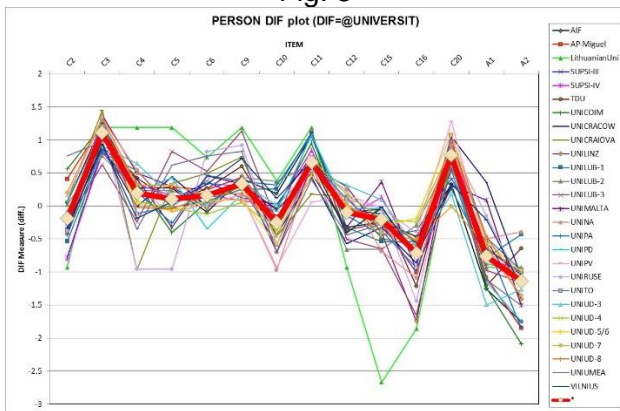
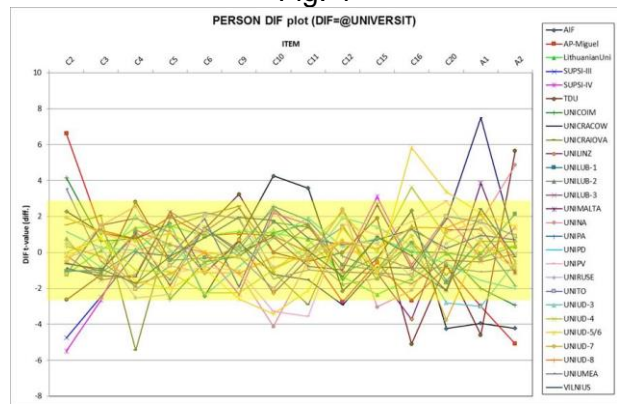


Fig. 4



For the locations: UNICRACOW, UNIUD-5/6, AIF, with the most evident DIF, the disattenuated correlation was always 1, meaning that the consideration of the specific difficulties for the location does not change the measures of the persons.

A third dimension that we do not consider in detail was formed by the items A4, A5, A6 and A7 which target the very specific aspect of degree of knowledge regarding the activities of a physic researcher. This dimension seems not to be relevant in the subsequent regression analysis of the choice to study physics.

**Cognitive and non-cognitive skills are interrelated**

Concerning the relationship between FACTOR A and FACTOR B, Figure 5 shows that this is positive with a disattenuated correlation of 0.64. Growing levels of both measures are associated to greater level of cognitive skill in physics (Tab. 1), approximated by the answers to the item C19 *Physics is the school subject I do best at now*. High levels of FACTOR A and FACTOR B are observed for students that decided to apply to study physics at university (question C24 of the SSQ

questionnaire). FACTOR A and FACTOR B seem moreover higher when the first interest for physics has begun at early stage of life (question C22) (Fig. 6). These facts suggest that investing in early childhood development, maybe relevant to increase the chance of having students with cognitive and non-cognitive skills well suited to undertake specific fields of training such as physics. These measures are moreover related to personal characteristics. Concerning GENDER (Table 1), Male have higher level of FACTOR A and FACTOR B than Female. Fig. 7 highlights the levels of FACTOR A and FACTOR B for the different survey locations: the students of UNILUB-3 show the highest level of FACTOR A, while that of SUPSI-IV the lowest one. For FACTOR B the students of UNIRUSE are at the top, while that of UNIUD-3 at the lowest level.

Fig. 5

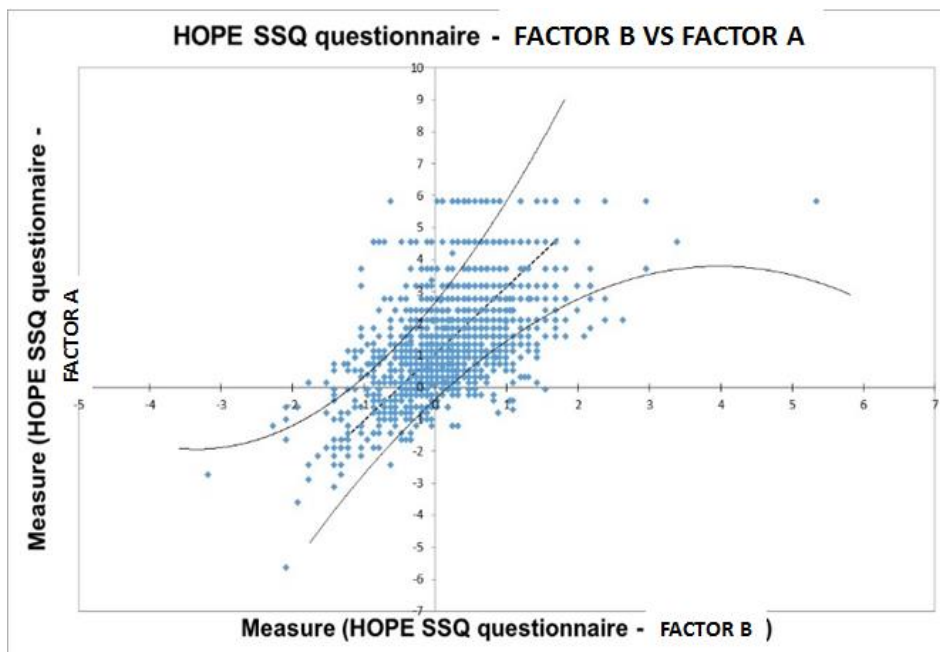


Table 1

Variable	Modality	FACTOR A			FACTOR B		
		Mean	S.E. mean	Pvalue(F-Test)	Mean	S.E. mean	Pvalue(F-Test)
GENDER	Male	1.69	0.06	0.000	0.30	0.03	0.000
	Female	1.26	0.09		0.16	0.04	
	Missing	1.17	0.09		-0.03	0.04	
Have you decided to apply to study physics at university?	No	1.23	0.05	0.000	0.14	0.02	0.000
	Yes	2.48	0.1		0.41	0.05	
	Missing	1.04	0.49		0.26	0.17	
Physics is the school subject I do best at now	1	0.27	0.11	0.000	-0.41	0.05	0.000
	2	0.82	0.09		-0.04	0.05	
	3	1.33	0.07		0.18	0.03	
	4	1.94	0.08		0.43	0.04	
	5	2.73	0.11		0.61	0.06	

Fig. 6

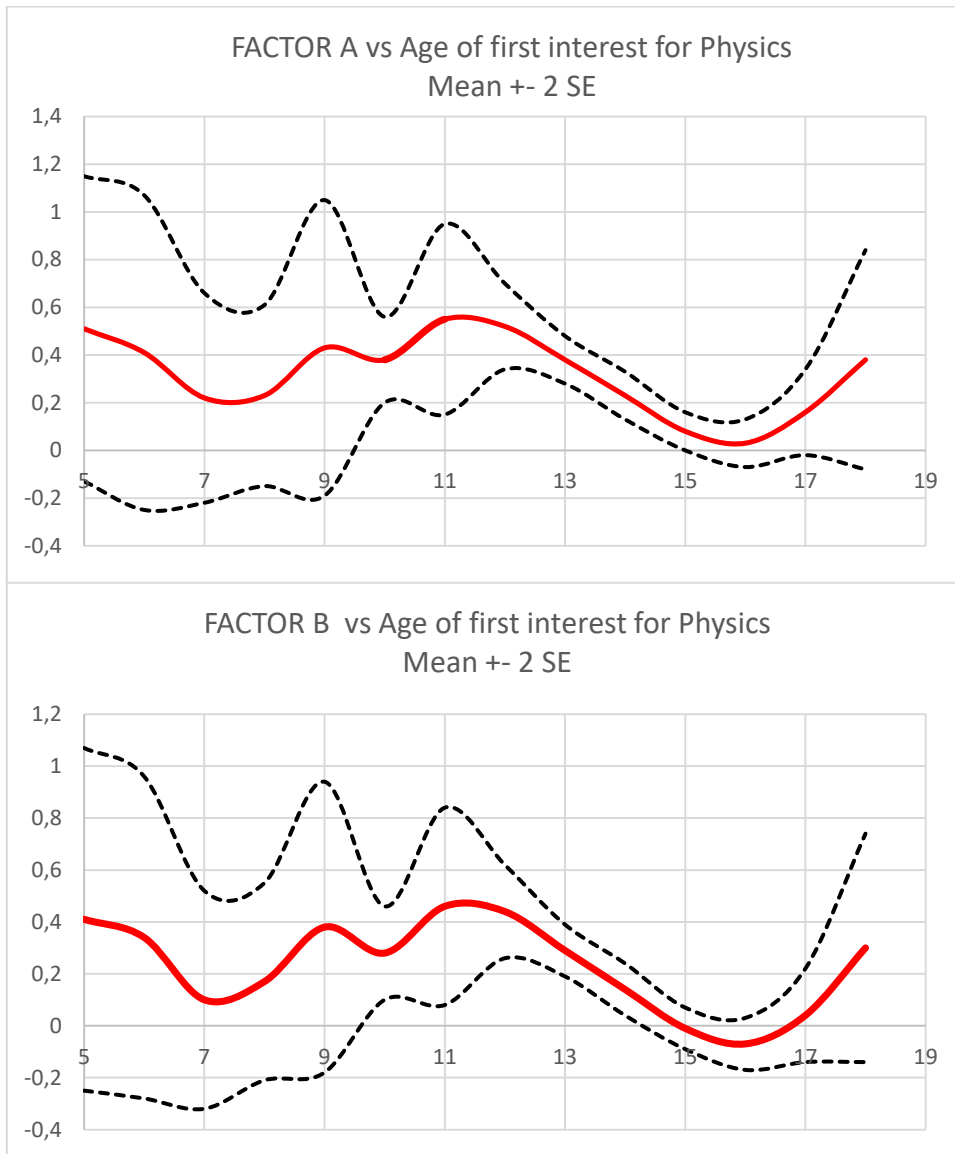
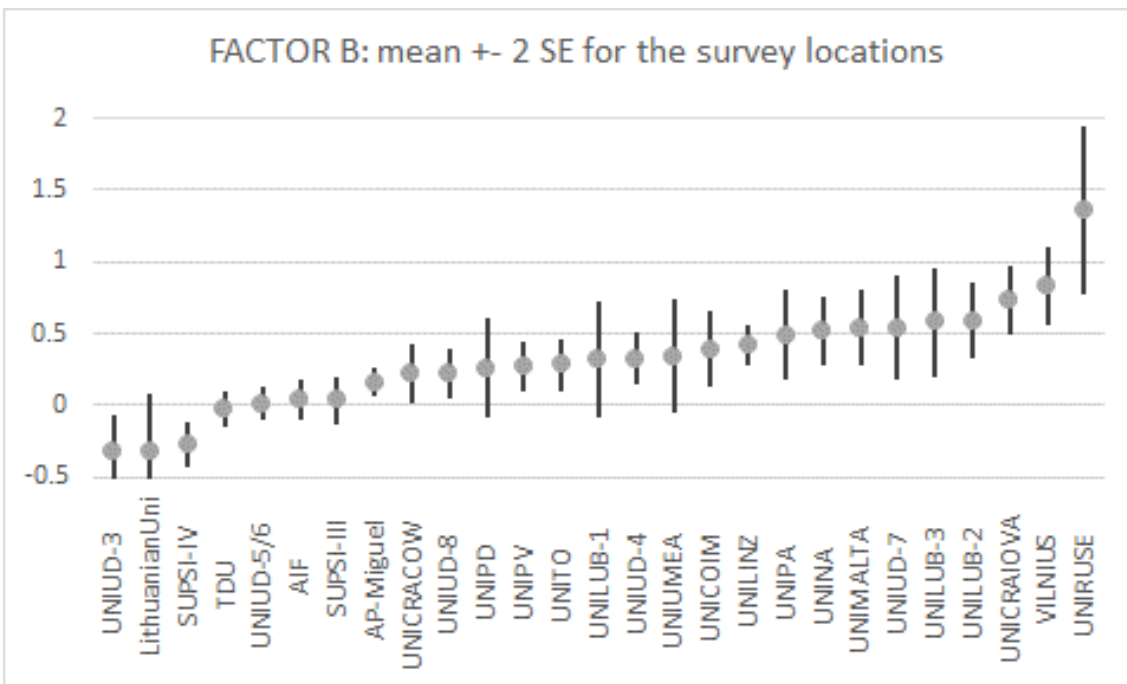
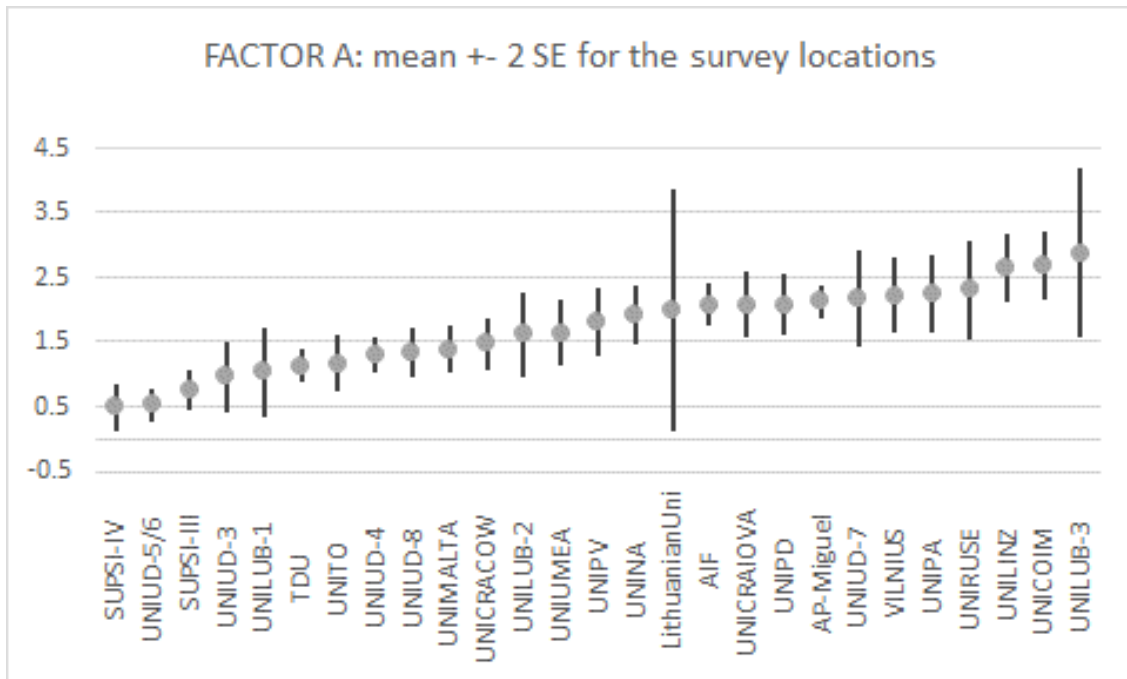


Fig. 7



## Explaining the choice of applying to study Physics

In order to understand the role of cognitive and non-cognitive skills, personal factors and site of interview questionnaire, in explaining the choice of applying to study physics, we applied a multilevel (mixed) logistic regression model, defined by:

$$Y_{ij} \approx \text{Bernoulli}(\pi_{ij})$$
$$\ln\left(\frac{\pi_{ij}}{1 - \pi_{ij}}\right) = \beta_0 + \beta_1 \cdot x_{1ij} + \beta_2 \cdot x_{2ij} + \dots + \beta_k \cdot x_{kij} + u_i$$
$$u_i \approx N(0, \sigma_u^2)$$

Where  $Y_{ij} = 1$ , if the student  $j$  of the site  $i$  choose to apply to study Physics,  $Y_{ij} = 0$ , otherwise,  $x_{rij}$  is the  $r$ -th explanatory variable,  $u_i$  is the effect of the  $i$ -th site. The explanatory variables considered here are MALE = 1 if male student, 0 otherwise, C19 = 1 to 5, depending on the answer to the question C19. *Physics is the school subject I do best at now*, C18 = 1 to 5 (PHYKNOW), depending on the answer to the question C18. *A wish to become a physics teacher* (TEACHER), FACTOR A and FACTOR B estimated above. The most common methods for estimating multilevel logistic models are based on likelihood. In this paper, we estimated the model using the R routine `glmer`, which is based on adaptive Gauss-Hermite approximations to the likelihood. However, having FACTOR A and FACTOR B estimated, they are, by definition, affected by error, and a straightforward estimation of the model would lead to inconsistent estimate of the coefficients (Griliches & Ringstad, 1970; Chesher, 1991). Among many other methods (Battauz *et al.*, 2011) the simulation and extrapolation method (SIMEX) by Cook and Stefanski (1994) has become a useful tool for correcting estimates in the presence of additive measurement error. The method is especially helpful for complex models with a simple measurement error structure. The R package `simex` (Lederer & Küchenhoff, 2006), provides functions to use the SIMEX method for various kinds of regression objects and to produce graphics and summary statistics for corrected objects. The SIMEX-method uses the relationship between the variance of the measurement error,  $\sigma_\varepsilon^2$  (estimated by the Rasch model) and the bias of the estimator when ignoring the measurement error, as a tool to get a consistent estimate of the coefficients. In particular, it uses the biased coefficients obtained applying the estimation methods without correction (naïve estimators), to a series of simulated data with inflated error variance (the SIM part of the algorithm) and a quadratic function fitted to explain the relationship between the coefficients and the error variance, to extrapolate the value of the coefficients when the error variance would be zero (the EX part of the algorithm). The naïve estimators were obtained applying the `glmer` function.

Just as descriptive tool, we applied a univariate logistic regression model for the dependent variable C24. *Have you decided to apply to study physics at university?* ( $Y_{ij} = 1$ , YES,  $Y_{ij} = 0$ , NO) against each single explanatory variable: as we can see from table 2 (A) all of them show a positive relation with the dependent variable. We then applied a multiple logistic regression model (without correction for errors in explanatory variables): from table 2 (B) we observe that the effect of variable MALE and FACTOR B is not statistically different from zero once taking account the effect of the other variables. In order to take account the effect of the site of interview, we estimated a multilevel logistic regression model: from table 2 (C) we may observe that again MALE and FACTOR B are not statistically significant and that the variance of the site effect is 1.003, a quite high value. If we compare the AIC index of the model (B) with that of the model (C) we see a reduction of almost 50 points that highlights

the goodness of the multilevel version of the model. So we drop the non-significant variables MALE and FACTOR B and we obtain the results of table 2 (D): the growth of the site effect is mainly due to the fact that for some site we do not have the information regarding the variable MALE, so that they were deleted from the regression model (C). Finally, we applied the SIMEX method to the multilevel logistic regression model, and we obtained the results of table 2 (E). As we may see the coefficient of ATTITUDE grows from 0.476 to 0.616 due to the effect of disattenuation related to measurement error, the other variables PHYKNOW and TEACHER, although still significant, reduce their effect being positive correlated with FACTOR A. Table 2 (F) shows the probability of choosing to study physics calculated for different values of FACTOR A (-5, -3, 0, 3, 5) and PHYKNOW and TEACHER (1,2,3,4,5), setting the site effect equal to 0 (the mean). As we may see a student with an FACTOR A at least 3 and a PHYKNOW and TEACHER at least 4 has a probability at least 0.90 of applying to study physics. A student with an FACTOR A of 5 has a probability of applying to study physics at least of 0.60 for every level of PHYKNOW and TEACHER.

We then investigated how the level of knowledge of physics is related to the latent variables FACTOR A and FACTOR B and other personal factors such as gender. To this end we defined the dichotomous variable  $C19 \geq 4$  ( $= 1$ ) if the person responded 4 or 5 to the question C19. *Physics is the school subject I do best at now*, 0 otherwise, and we estimated the logistic regression models reported in table 3. As we may see, all models tell us the same story: the effect of MALE, FACTOR A and FACTOR B are significantly different from zero and positive related to the event that the person has high levels of cognitive skills in physics. So male tend to have greater level of cognitive skills than female, for every given levels of FACTOR A and FACTOR B. Looking at the size of the coefficients FACTOR B has a greater positive effect on cognitive skills than FACTOR A. So FACTOR B, although not statistically significant FACTOR in explain the choice to study physics, indirectly influence such choice by its positive effect on cognitive skills, that are positive related to this choice. As we may see, comparing model of Tab. 3 (A) and Tab. 3 (C), the site effect is not relevant: AIC does not change much adding the random intercept, and the estimated variance of the site effect is small (only 0.098). Looking at Tab. 3 (A) and Tab. 3 (B) we may see that the coefficients of FACTOR A and FACTOR B grow, once we take into account their measurement error by SIMEX method, while the effect of MALE decrease a little. Tab. 3 (E) and (F) report the probability of the event ( $C19 \geq 4$ ) given the level of the explanatory variables, estimated from the coefficients of Tab. 3 (B). As we may see, a male with level 2 of FACTOR A and FACTOR B has a probability of 0.800 of declaring ( $C19 \geq 4$ ), while a female with the same level of FACTOR A and FACTOR B has a probability of 0.568 of declaring ( $C19 \geq 4$ ). Moreover we may see that, given a male with level 0 of FACTOR A and FACTOR B, the probability rise to 0.737 when FACTOR A grows by 2 points, while the probability rise to 0.850 when FACTOR B grows by 2 points.

Finally we investigated how the wish to become a physics teacher, is related to the latent variables FACTOR A and FACTOR B and gender. To this end we defined the dichotomous variable  $C18 \geq 4$  ( $= 1$ ) if the person responded 4 or 5 to the question C18. *I wish to become a physics teacher*, 0 otherwise, and we estimated the logistic regression models reported in table 4. Gender was not statistically significant so we excluded this variable from the models. The multilevel version did not improve the explanation of the variability of the data (the AIC increased and the estimate of the variance of the site effect was almost zero) so our final choice was for model Tab.4 (B). We may see both FACTOR A and FACTOR B are positively correlated with the wish to become a physics teacher. We observe that the effect of FACTOR A is stronger than that of FACTOR B, and also in this case the SIMEX correction is relevant, changing the order of the size of the effects. From table (E) we may see that in order to reach an appreciable level of the probability of declaring 4 or 5 to the question C18. *I wish to become a physics teacher*, the levels of FACTOR A and FACTOR B must be very high.



Tab. 2 – Logistic regression models for the Choice to study physics

(A)					(B)					
Simple logistic regression models of Choice to study physics VS					Multiple Logistic regression model of Choice to study physics VS					
VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE	VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE	
MALE	0.357	0.191	1.875	0.061	(Intercept)	-3.9364	0.391	-10.068	0.000	
TEACHER	0.444	0.056	7.925	0.000	MALE	0.1578	0.226	0.698	0.485	
PHYKNOW	0.425	0.058	7.270	0.000	TEACHER	0.1971	0.081	2.434	0.015	
FACTOR A	0.474	0.046	10.220	0.000	PHYKNOW	0.2579	0.093	2.778	0.005	
FACTOR B	0.462	0.093	4.940	0.000	FACTOR A	0.4921	0.069	7.116	0.000	
					FACTOR B	-0.2007	0.144	-1.396	0.163	
					AIC	750.7				
(C)					(D)					
Multilevel Logistic regression model of Choice to study physics VS					Multilevel Logistic regression model of Choice to study physics VS					
VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE	VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE	
(Intercept)	-3.988	0.481	-8.289	< 2e-16	(Intercept)	-3.877	0.392	-9.901	< 2e-16	
MALE	0.111	0.244	0.457	0.648	TEACHER	0.211	0.071	2.966	0.003	
TEACHER	0.176	0.088	2.008	0.045	PHYKNOW	0.222	0.077	2.883	0.004	
PHYKNOW	0.241	0.098	2.459	0.014	FACTOR A	0.476	0.062	7.7	0.000	
FACTOR A	0.519	0.077	6.741	0.000	Sigma2u	1.383				
FACTOR B	-0.235	0.160	-1.465	0.143						
Sigma2u	1.003									
AIC	707.8									
(E)					(F)					
SIMEX - Multilevel Logistic regression model of Choice to study physics VS					Probability of choosing to study physics given the level of the explanatory variables (based on model ( E ))					
VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE	FACTOR A					
(Intercept)	-3.874	0.400	-9.695	< 2e-16	TEAC/PHY	-5	-3	0	3	5
TEACHER	0.166	0.074	2.245	0.025	1	0.004	0.012	0.073	0.333	0.631
PHYKNOW	0.165	0.080	2.061	0.040	2	0.010	0.033	0.176	0.576	0.823
FACTOR A	0.616	0.079	7.787	0.000	3	0.026	0.084	0.367	0.787	0.927
Sigma2u	1.450				4	0.068	0.199	0.612	0.909	0.972
					5	0.165	0.403	0.811	0.965	0.989

Tab. 3 - Logistic regression models for the event (C19>=4)

(A)						(B)							
Multiple Logistic regression model of C19 >= 4 VS						SIMEX - Multiple Logistic regression model of C19 >= 4 VS VS							
VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE		VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE			
(Intercept)	-1.219	0.160	-7.599	0.000		(Intercept)	-1.328	0.171	-7.755	0.000			
MALE	0.360	0.163	2.206	0.027		MALE	0.341	0.167	2.048	0.041			
FACTOR A	0.444	0.059	7.525	0.000		FACTOR A	0.505	0.073	6.883	0.000			
FACTOR B	0.615	0.124	4.956	0.000		FACTOR B	0.680	0.157	4.329	0.000			
AIC	1123.3												
(C)						(D)							
Multilevel Logistic regression model of C19 >= 4 VSs VS						SIMEX - Multilevel Logistic regression model of C19 >= 4 VS VS							
VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE		VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE			
(Intercept)	-1.141	0.187	-6.108	1.01E-09		(Intercept)	-1.255	0.195	-6.423	2.13E-10			
MALE	0.345	0.167	2.069	0.039		MALE	0.328	0.170	1.928	0.054			
FACTOR A	0.410	0.062	6.613	0.000		FACTOR A	0.462	0.079	5.861	0.000			
FACTOR B	0.672	0.131	5.125	0.000		FACTOR B	0.764	0.169	4.518	0.000			
Sigma2u	0.098					Sigma2u	0.084						
AIC	1121.5												
(E)						(F)							
Probability of the event (C19>=4) given the level of the explanatory variables MALE (based on model (B))						Probability of the event (C19>=4) given the level of the explanatory variables FEMALE (based on model (B))							
		FACTOR B							FACTOR B				
	MALE	-4	-2	0	2	4		FEMALE	-4	-2	0	2	4
FACTOR A	-4	0.003	0.013	0.047	0.162	0.430	FACTOR A	-4	0.002	0.009	0.034	0.115	0.305
	-2	0.009	0.034	0.120	0.346	0.674		-2	0.006	0.024	0.085	0.246	0.479
	0	0.024	0.087	0.272	0.593	0.850		0	0.017	0.062	0.193	0.421	0.604
	2	0.063	0.208	0.506	0.800	0.940		2	0.045	0.148	0.360	0.568	0.668
	4	0.156	0.419	0.737	0.916	0.977		4	0.111	0.298	0.524	0.651	0.695

Tab. 4 - Logistic regression models for the event (C18>=4)

(A)					(B)				
Multiple Logistic regression model of C18 >= 4 VS					SIMEX - Multiple Logistic regression model of C18 >= 4 VS				
VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE	VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE
(Intercept)	-2.803	0.155	-18.054	< 2e-16	(Intercept)	-2.957	0.178	-16.597	< 2e-16
FACTOR A	0.342	0.062	5.504	0.000	FACTOR A	0.421	0.079	5.316	0.000
FACTOR B	0.393	0.133	2.946	0.003	FACTOR B	0.393	0.164	2.391	0.017
AIC	845.1								
(C)					(D)				
Multilevel Logistic regression model of C18 >= 4 VS					SIMEX - Multilevel Logistic regression model of C18 >= 4 VS				
VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE	VARIABLES	ESTIMATES	S.E.	Z-VALUE	P-VALUE
(Intercept)	-2.803	0.155	-18.054	< 2e-16	(Intercept)	-2.956	0.178	-16.611	< 2e-16
FACTOR A	0.342	0.062	5.504	0.000	FACTOR A	0.422	0.079	5.348	0.000
FACTOR B	0.393	0.133	2.946	0.003	FACTOR B	0.385	0.164	2.344	0.019
Sigma2u	0.000				Sigma2u	0.000			
AIC	847.1								
(E)									
Probability of the event (C19>=4) given the level of the explanatory variables (based on model (B))									
		FACTOR B							
		-4	-2	0	2	4			
FACTOR A	-4	0.002	0.004	0.010	0.021	0.044			
	-2	0.005	0.010	0.022	0.047	0.097			
	0	0.011	0.023	0.049	0.102	0.200			
	2	0.024	0.052	0.108	0.209	0.367			
	4	0.055	0.113	0.219	0.381	0.574			

## Conclusions

We Rasch analyzed the data from the HOPE-SSQ questionnaire, and we identified two latent traits, FACTOR A and FACTOR B. These traits possess good psychometric properties given the optimal fit to the Rasch model, in particular the Rating Scale Model (Andrich, 1978). Their external validity is moreover confirmed by the correlation with important aspects and personal characteristics. FACTOR A, in particular, characterized by curiosity for the world and interest in its comprehension, seems to be higher when the first interest for physics starts at early childhood, and it is higher for males. FACTOR B is characterized by the wish to get an interesting job, enhancing employment prospects, stimulus from visiting museums, laboratories, seeing things on the TV or internet, learning interesting things at schools, receiving encouragement from parent and friend and information during visit from university staff. FACTOR A and FACTOR B are positively correlated and we suppose that the mechanisms that enhance FACTOR B have a greater effect on students with high levels of FACTOR A. We may think that FACTOR B enhances FACTOR A, but only if this is at certain level: it is difficult to motivate a student that is not curious about the world around him. Under this respect, we suppose that FACTOR A is a precondition, given also its development in early childhood, and as Borghans et al. (2008) suggest for other personal traits, it may be related "to neural substrates and biological factors". We used the measures of these traits for each person, as explanatory variables, among the others, of important facts. First of all the decision to study physics at university. We applied a multilevel logistic regression model, with SIMEX correction, to explain such event and we found that only the FACTOR A, the level of knowledge of physics, and the wish to become a physic teacher are positively and significantly correlated with the choice to study physics. Moreover we observed a significantly effect of the location of the interview. FACTOR B does not account, instead, for choice given the other explanatory variables. Here we underline that the level of cognitive skills (knowledge of physics) is an important factor in determining the choice, as we observed in other contexts (Battauz, 2006). Enhancing the level of knowledge of physics may therefore lead to increase the number of students that chose to study physics. Obviously, there is a great interrelation between the factors: indeed a second logistic regression model showed us that FACTOR A and FACTOR B are positively and significantly correlated with the level of knowledge of physics; in this case, also the gender explains this level, being males advantaged with respect to females. No site effect, instead, were observed for this relation. Obviously, also other factors, such as teachers, classes and schools, explain the cognitive skills, but we do not have this information here. These results tell us that in order to increase the level of knowledge of physics, we may use the factors that enhance FACTOR B, but just as for the gender variable, if FACTOR A would rely on genetic factors, the only way to stimulate knowledge of physics by FACTOR A would be to select, at early age, the students with higher FACTOR A and dedicate to them special educational programs. Finally, a third model, showed to us that the wish to become a physic teacher is positively correlated to FACTOR A and FACTOR B, but not to gender and sites. Therefore, we may say that, although FACTOR B does not explain the choice to study physics, it explains, with FACTOR A, the other important factors, which explain this choice: cognitive skills in physics and the wish to become a physic teacher. We think that the results obtained in this paper may give a contribution to the discussion and some suggestions regarding the policies to adopt in order to increase the access of students to physics studies.

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