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Voting in Small Committees

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Abstract

We analyze the voting behavior of a small committee that has to approve or reject a project proposal whose return is uncertain. Members have diverse preferences: some of them want to maximize the expected value, while others have a bias towards project approval and ignore their information on the project value. We focus on the most efficient use of scarce information when members cannot communicate prior to voting, and we provide insights on the optimal composition of the committee. Our main result is that the presence of biased members can improve the voting outcome, by simplifying the strategies of unbiased members. Thus, committees with diverse members perform as well as homogeneous committees, and even better in some cases. In particular, when value-maximizing members outnumber biased member by one vote, the optimal equilibrium becomes unique.

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1 Introduction

In many committees, members are nominated by (and thus represent the interests of) different institutions and this reflects in different voting behaviors. Consider, for instance, boards

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of directors whose objective, in principle, is to maximize firm value. Directors represent different stakeholders, (majority and minority shareholders, investors, workers, etc.) whose objectives may not be aligned. Another example¹ is provided by monetary policy committees, where some members are chosen within the staff of the central bank while other members are appointed by external bodies, such as the Government (the Bank of England Monetary Policy Committee is a typical example). In this case, internal members are usually more concerned about inflation while external members are more concerned about unemployment. In general, empirical studies show that members belonging to different groups have significant differences in their voting behaviors and that these differences can be explained by factors such as political pressure or the channel of appointment, especially when committee members face retention decisions (see, for example, Sheperd [2009], and Harris, Levine and Spencer [2011]).

The present paper analyzes the effect of member heterogeneity by studying the voting behavior of a small committee that has to approve or reject a project. We consider two types of players: expected value maximizers and biased members who always vote in favor of the project, even disregarding their private information. Then, the following question arises: why should biased members be allowed on this committee? Our model shows that their presence is beneficial in the absence of pre-voting communication because it ensures uniqueness and optimality of the equilibrium strategy profile. The intuition is that the bias provides certainty about some members' strategies thus simplifying the responses of the others, and therefore reducing the number of (otherwise) multiple equilibria. In particular, we explore the behavior of uninformed value-maximizing members. Given that they want to maximize the probability that the committee makes the correct decision, they face the question of how to avoid influencing the decision and let informed members determine it. The equilibrium voting strategies prescribe that uninformed unbiased members systematically contrast the vote of biased members. Indeed, in many small committees dissent voting is commonly observed (Spencer [2006]). On the basis of the equilibrium voting strategies, we determine the optimal composition of the committee, consisting in letting unbiased members outnumber biased members by just one vote. Our result is consistent with the actual composition of some committees such as the Bank of England Monetary Policy Committee or the Italian Constitutional Court.

The paper is organized as follows. Section 2 reviews the main literature. Section 3 presents the basic model. Section 4 examines, as a benchmark, the voting game in a committee composed only of value-maximizing members. Section 5 introduces biased members and

 $^{^{1}}$ Additional examples are special juries as Supreme or Constitutional Courts, and technical committees, where politicians, bureaucrats and experts meet to provide advice.

analyzes if and how results change when members have different objectives. Then, in Section 6 we discuss some assumptions of the model. Finally, Section 7 concludes. All proofs are collected in the Appendix.

2 Related literature

Since Condorcet's seminal contribution, namely his Jury Theorem, the literature about voting has been constantly growing. A lot of papers have generalized the Jury Theorem², and many others have extended voting games to include both naive and strategic voting³.

Traditionally, in this literature the aim of voters has been to aggregate information, with the assumption that taking the right decision (that is, guessing the correct state of the world) was the common objective of all the players. In fact, we believe this is not the case, as heterogeneity of preferences is well documented both in large and in small elections⁴. Feddersen and Pesendorfer [1996; 1997] have focused on heterogeneity in large elections, showing that full information aggregation is still possible. In particular, they show that the probability of electing the "wrong" candidate asymptotically goes to zero.

Since the seminal contribution by Austen-Smith [1990], information sharing and voting in small committees has been increasingly analyzed along with the possibility of communication⁵. Nonetheless, in small committees, the presence of preference heterogeneity and (possible) resulting conflicts of interests appear to be a relevant problem, as information sharing and aggregation may be severely limited by strategic behavior (see also Gerling et al., 2005).

Thus, one possibility is to look for optimal voting rules to minimize information losses. In a standard Condorcet Jury Theorem framework, Chwe [1999] suggests to provide minority members with optimal incentives to participate in voting, in order not to waste their information. Things become more complicated when the relevant issue is not to find optimal voting rules but, rather, an optimal way to aggregate "relevant" information⁶. Li, Rosen and Suen [2001] examine a two-person committee where each member receives a private signal and reports his information. Since members have conflicting interests, strategic considera-

²See, for instance, Duggan and Martinelli [2001], Myerson [1998] and McLennan [1998]. See also Piketty [1999] for a brief review of recent contributions about the information-aggregation role of political institutions.

 $^{^{3}}$ See Austen-Smith and Banks [1996] and an experiment on the use of strategic voting by Eckel and Holt [1989].

⁴See for instance Blinder [2006], Jung [2011], Riboni and Ruge-Murcia [2007] for the case on Monetary Policy Committees.

⁵This strand of literature has been reviewed by Austen-Smith and Feddersen [2009]. See also Adams and Ferreira [2007], Harris and Raviv [2008], and Raheja [2005] for communication in boards of directors.

⁶Berk and Bierut [2004] suggest that often in small committees it is technologically or politically unfeasible to implement optimal voting rules, thus binding voting to be based on simple majority.

tions induce information misreporting and there is no truth-telling equilibrium. Conflict of interest prevents full information aggregation also in larger committees, as shown by Maug and Yilmaz [2002]. These authors suggest to group voters into two separate classes because such a voting mechanism may alleviate the incentive to withhold information when voters have strong conflicts of interests and individual information differs. Thus, voting decisions become more informative. Examining a committee of experts, Wolinsky [2002] suggests to solve the problem in a similar way, by partitioning members in different groups⁷.

We take a different approach to the problem of information sharing among committee members by examining how the voting outcome can be optimized when members do not communicate. Without imposing any explicit revelation mechanism and without looking for optimal voting rules or protocols, we suggest an optimal composition rule as a sufficient device to provide the highest possible level of information aggregation, even when biased members do not use their private information. The positive role of the latter in our model relies on the fact that their presence on the committee eliminates multiple (and possibly suboptimal) equilibria.

3 The model

A committee is composed of $2n + 1, n \ge 1$, members who have to decide by majority vote whether to approve a project (voting "yes") or reject it (voting "no"). If the proposal is rejected, a value of 0 is realized. When accepted, the project yields value v = -1 if the state of the world is low (L), and v = 1 if the state of the world is high (H). Thus, $v : \{-1, 1\}$.

Each state, and thus each value, has the same prior (i.e., $\frac{1}{2}$). This implies that when members have no information on the state of the world, there is no one choice that dominates the other. Given these probabilities, the highest expected value that can be achieved by voting correctly (rejecting the project in L and approving it in H) is $\frac{1}{2}$. Note that a single uninformed decision maker would always obtain an expected value of 0.

We consider a simple information structure where, before voting, any member of the committee learns the true state with probability $\alpha \in [\frac{1}{2}, 1)$ and learns nothing with probability $1 - \alpha$.⁸ As a consequence, the information set of a generic member *i* is $\Omega_i = \{\omega_i\}$, with $\omega_i \in \{H; L\}$, when *i* is informed, while it is $\Omega_i = \{H, L\}$, when *i* does not know the true state of the world. The probability of being informed is identically and independently distributed

⁷Committee members have heterogeneous preferences also in the model of Cai [2009]. The focus of Cai, however, is the optimal size of the committee rather than its composition.

⁸Alternatively, we can assume that every member observes a signal that is perfectly informative with probability α and is totally noisy with probability $1 - \alpha$. Referring to this interpretation, it can be shown that our results would not qualitatively change if the signal was only partially noisy (Balduzzi [2005]).

across members. Each member does not know who else is informed. Furthermore, we consider the case in which committee members can become informed at no cost. As we point out at the end of section 4, introducing a fixed cost for acquiring information may set an upper bound to the optimal size of the committee.

We assume that members cannot communicate prior to voting, and as usual in the literature on committee voting, we do not consider abstention⁹. Both assumptions are discussed in Section 6. Given that abstention is not allowed, the action set of each player has only two elements: vote "yes" to accept the project, and "no" to reject it. A strategy s_i is a member *i*'s voting behavior, conditional on his information set. A mixed strategy is defined as the probability that a member votes "yes".

The committee is composed of unbiased members who want to maximize the expected value of the project, E(v), and of biased members who want to approve the project independently of the state of the world. We assume that all members are risk neutral and that their types are common knowledge. Let M denote value-maximizing members, and B members with a bias. Then, we call m and b the probabilities of voting "yes" for an *uninformed* member of type M and B respectively.

The utility function of an M type can be directly expressed in terms of the expected value of the project: $u_M(E(v)) = E(v)$. Then, an M member will choose the strategy that maximizes E(v). Notice that, given the values the project can take, maximizing E(v) is equivalent to maximizing the probability that the committee takes the correct decision. Indeed, the latter is equal to the sum of the probabilities that "yes" wins when the actual value of the alternative is 1 and that "no" wins when the actual value of the alternative is -1:

$$\frac{1}{2} \left\{ Y(\cdot \mid v = 1) + [1 - Y(\cdot \mid v = -1)] \right\},\$$

where the function $Y(\cdot|\cdot)$ is the conditional probability that the board as a whole votes "yes". The expected value of the project is:

$$E(v) = \frac{1}{2} [1Y(\cdot | v = 1) - 1Y(\cdot | v = -1)]$$

and it is straightforward to notice that the two expressions are strategically equivalent.

Given that a member can influence the outcome (and consequently his own utility) only when he is pivotal, conditioning the vote on being pivotal is a weakly dominant strategy for an M member who wants to maximize E(v). We concentrate only on equilibria where

⁹Of course we acknowledge some exceptions, such as Morton and Tyran (2008). Note also that in many committees abstention is explicitly or implicitly ruled out (juries, the European Courts of Human Rights and the Italian Constitutional Court are some examples).

the M members choose such strategies.¹⁰ The solution concept we use is Bayesian Nash Equilibrium.

The utility function of a B member positively depends on the approval of the project.¹¹ A B member always supports the project, regardless of the value which is *ex post* realized. His utility u_B , therefore, depends on the final decision of the committee, and can take the following two values: $u_B = 1$ if the project is approved, and $u_B = 0$ if the project is rejected. This clearly implies that always voting "yes" is a dominant strategy for a B member. For simplicity, we abstract from additional problems, such as a B member's potential loss of reputation when the approval of the project creates a loss.

Finally, before analyzing the voting behavior of the committee, we introduce the definition of compensating strategy that will be useful in the following sections.

Definition 1 (Compensating strategy) Two members are playing compensating strategies when the following conditions are jointly satisfied: i) they are both uninformed; ii) they play "yes" with probabilities whose sum is equal to 1. When these probabilities take extreme values (0 and 1), compensation is in pure strategies.

Note that this definition describes strategies but does not require members to know who is informed and who is not. Compensation is an *ex ante* concept. It may well happen that one of the two individuals who compensate when uninformed is in fact informed, so that compensation does not necessarily obtain *ex post*.

4 The benchmark

We define our benchmark as a committee only composed of members who want to maximize E(v), i.e. members of type M. Clearly, whenever an M member is informed, he votes according to his information. The issue is to define what an uninformed M member should do. Intuitively, any uninformed member has an incentive to leave the final decision to the others, who may be informed. It can then be shown that there are four types of equilibria differing as to the behavior of uninformed members.

The first consists of an (asymmetric) equilibrium, where all but one member compensate for each other in pure strategies when uninformed, while the remaining member plays any strategy (when uninformed). Given that one of the members can actually play "yes" with any probability, and that the identity of those members who vote "yes" and those who vote

¹⁰This rules out "uninteresting" equilibria where each member is never pivotal. For instance when every member votes "yes" independently of his information. However it can be easily verified that all these equilibria yield E(v) = 0.

¹¹Obviously, nothing substantial in our results would change if a biased member always supported rejection.

"no" (as well as that of the member who possibly uses a mixed strategy) is interchangeable, there exists in fact a multiplicity of such equilibria all of which yield the same expected value. Considering that members condition their strategies on being pivotal, no symmetric equilibrium in pure strategies is possible¹². The second consists of a (symmetric) unique equilibrium where all the members compensate for each other in mixed strategies, voting "yes" with probability 1/2. The third consists of an equilibrium where an even number of uninformed members compensate for each other in pure strategies, while the remaining members compensate in symmetric mixed strategies, voting "yes" with probability 1/2. Finally, the last consists of an equilibrium where there are more members voting "yes" ("no") than members voting "no" ("yes"), while the others choose the same mixed strategy, voting "yes" with probability lower (greater) than 1/2. Again there is a multiplicity of equilibria of the third and fourth type because the identity of the players is interchangeable and because there can be different numbers of players choosing mixed strategies.

In order to better understand the nature of the first type of equilibria, consider a simple example with $\alpha = \frac{1}{2}$ and n = 2, so that there are five members M_i , i = 1, 2, 3, 4, 5. Suppose that four members vote "yes" when uninformed. The remaining member knows that he is pivotal only if two members vote "no". But, given the above strategies, this happens only if the two members who vote "no" are in fact informed. Then, the remaining member should vote "no". This tells us that (as members condition their strategies on being pivotal) there cannot exist a symmetric equilibrium in pure strategies where everybody votes "yes" (neither, by the same argument, an equilibrium where everybody votes "no"). Moreover, it immediately appears that a situation where four members vote "yes" (or "no"), when uninformed, cannot be an equilibrium: voting "yes" ("no") is not the best response for an uninformed individual when there are already three members following the "yes" ("no") strategy. His best response is to vote the opposite of another uninformed member, thus giving rise to the asymmetric equilibrium. Once there are three members voting "yes" and two members voting "no", nobody has an incentive to change his strategy. Consider one of those member voting "yes". The probability that he is pivotal is the same in both states, so he is indifferent between voting "yes" or "no".

The expected value of these equilibria can be easily computed for $\alpha = 1/2$. In the case where two members vote "no" when uninformed, e.g. $m_1 = m_2 = 0$, and $m_3 = m_4 = m_5 = 1$, the expected value is

$$E(v)^{PS} = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] = \frac{1}{2}\left[1 - \frac{1}{8}\right] = \frac{7}{16} < \frac{1}{2}$$

 $^{^{12}\}mathrm{On}$ this point, see footnote 10.

which is clearly equal to the value that obtains if three members vote "yes" when uninformed, e.g. $m_1 = m_2 = m_3 = 0$; and $m_4 = m_5 = 1$

$$E(v)^{PS} = \frac{1}{2} [Y(\cdot|v=1) - Y(\cdot|v=-1)] = \frac{1}{2} \left[\frac{7}{8} - 0\right] = \frac{7}{16} < \frac{1}{2}.$$

Moreover it can be easily checked that E(v) does not change even if there is one member choosing a mixed strategy. For example when $m_1 = m_2 = 0$, $m_3 = m_4 = 1$ and $m_5 = 1/2$

$$\begin{split} E(v)^{PS} &= \frac{1}{2} [Y(\cdot | v = 1) - Y(\cdot | v = -1)] = \\ \frac{1}{2} [1 - [1 - Y(\cdot | v = 1)] - Y(\cdot | v = -1)] = \\ \frac{1}{2} \left[1 - 2\frac{1}{16} \right] = \frac{7}{16} < \frac{1}{2}. \end{split}$$

Notice that, in the spirit of Condorcet, the expected value is bigger than the value obtained by a single decision maker (0). Still, this committee cannot provide full information (yielding $E(v)^{FI} = \frac{1}{2}$) as it may collectively be uninformed. In other words, the committee does not always make the correct decision. When all the members are uninformed, the decision (whatever it is) is correct with probability 1/2. Moreover, the decision is wrong with probability 1/2 if the only informed members are those voting according to the actual state even if uninformed (those choosing $m_i = 1$ if uninformed when the actual state is v = 1, or those choosing $m_i = 0$ if uninformed when the actual state is v = 0).

Consider now the second type of equilibrium where all the uninformed members compensate for each other in (symmetric) mixed strategies. Intuitively, in this equilibrium any member randomizes as long as he is pivotal in both states of the world with the same probability, given the other members' strategies. But this is simultaneously true for any single member, only if all the members are pivotal in each state of the world with the same probability. The only profile which is compatible with this logic is then the one where all the members compensate for each other in mixed strategies, voting "yes" with probability 1/2. This argument rules out any other possible equilibrium in mixed strategies: whenever a member is not indifferent between voting "yes" or "no" (because the probability that he is pivotal in a state is higher than the probability that he is pivotal in another state), he plays a pure strategy. But then, other members will have an incentive to deviate from any mixed strategy to compensate for his pure strategy. Again, we can easily compute the expected value for the five member case with $m_1 = m_2 = m_3 = m_4 = m_5 = \frac{1}{2}$ and $\alpha = \frac{1}{2}$, obtaining

$$E(v)^{MS} = \frac{1}{2} [Y(\cdot|v=1) - Y(\cdot|v=-1)] = \frac{1}{2} [1 - [Y(\cdot|v=-1) + (1 - Y(\cdot|v=1))] = \frac{1}{2} \left[1 - \frac{53}{256}\right] = \frac{203}{512} < \frac{7}{16}$$

where $[Y(\cdot|v=-1) + (1 - Y(\cdot|v=1))]$ is the probability of making the wrong decision. Thus, the expected value falls short of that obtainable in the pure strategy equilibrium.

The third type of equilibrium looks like a combination of the previous two. In terms of expected value, these equilibria perform better than the mixed strategy ones but worse than equilibria with compensation in pure strategies. The expected value for the five member case with $m_1 = 1, m_2 = 0, m_3 = m_4 = m_5 = \frac{1}{2}$ and $\alpha = \frac{1}{2}$, is

$$E(v)^{MPS} = \frac{1}{2} [Y(\cdot|v=1) - Y(\cdot|v=-1)] = \frac{1}{2} [1 - [Y(\cdot|v=-1) + (1 - Y(\cdot|v=1))]] = \frac{1}{2} [1 - .171874] = .414063; \qquad \frac{203}{512} < .414063 < \frac{7}{16}$$

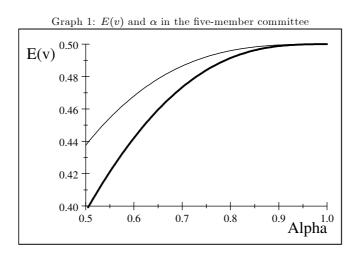
These mixed equilibria also dominate equilibria of the fourth type where there are more members voting "yes" ("no") than members voting "no" ("yes"), while the remaining members choose the same mixed strategy, voting "yes" with probability lower (greater) than 1/2. An example of the fourth type of equilibrium is found in the proof of the following Proposition, which generalizes our findings.

Proposition 1 (Benchmark) In a voting game with 2n + 1 members of type M, informed members always play according to their information. There exist four types of equilibria differing as to the behavior of uninformed members: i) multiple equilibria where n members always vote "no", n members always vote "yes", and one member chooses $m \in [0,1]$; ii) a unique equilibrium in mixed strategies where 2n + 1 members randomize with probability $\frac{1}{2}$; iii) multiple equilibria where 2n - 2k, k = 1, 2...n - 1, members compensate in pure strategies and 2k + 1 members play $m_k = 1/2$; iv) multiple equilibria where $(n - k_1)$ members choose $m_j = 0$ ($m_z = 1$) and ($n - k_2$) members choose $m_z = 1$ ($m_j = 0$) with $k_1 < k_2 \le n$ while all the others, $k \ne j, z$, choose the same mixed strategy $m_k > \frac{1}{2}$ ($m_k < \frac{1}{2}$). Equilibria of type i) yield expected value $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$. All other equilibria yield a lower expected value. The intuition for this result is the same as in the five-member case: uninformed members do not want to influence the outcome of the voting process, so they compensate for each other, and leave the final decision to possibly informed members. This is also what happens in the second kind of equilibria where uninformed players compensate for each other in mixed strategies. Compensation is more effective when played in pure strategies, as it is realized with probability one: E(v) is higher in the equilibria where 2n agents compensate in pure strategies than in the symmetric equilibrium in mixed strategies or in the equilibria where only part of the members choose pure strategies while the others choose mixed strategies. Then, in what follows we take the level of $E(v)^*$ as our benchmark.

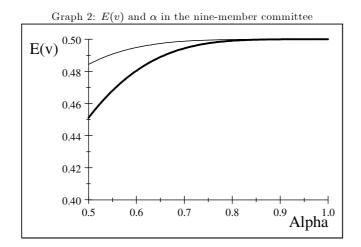
Definition 2 Any equilibrium that yields $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$ is defined optimal.

Notice that $(1-\alpha)^{n+1}$ is the probability that the decision is wrong, given by the probability that all the members are uninformed, plus the probability that the only informed members are those voting according to the actual state even if uninformed (those choosing $m_i = 1$ if uninformed when the actual state is v = 1, or those choosing $m_i = 0$ if uninformed when the actual state is v = -1).¹³

Just for illustration, we draw in Graph 1 and 2 the relationship between E(v) and the probability of having informed members (α), in committees with five and nine members. In both graphs, we compare the optimal equilibrium outcome (thin line) with the symmetric mixed strategy equilibrium outcome (thick line).



¹³That $(1-\alpha)^{n+1}$ represents the probability of making the wrong decision is shown in the proof of Proposition 1, point v).



The graphs show a positive relationship between E(v) and α in both equilibria. They also show that the mixed strategy equilibrium never yields a higher expected value than the one with compensation in pure strategies. When all the members are informed ($\alpha = 1$), there is no difference between the two equilibria and E(v) is the same. It is also clear that E(v) is growing in n. These relations are formalized by Corollary 1 which immediately follows from $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$

Corollary 1 When an optimal equilibrium is played, $E(v)^*$ is increasing both in n and in α at a decreasing rate.

The positive relation between $E(v)^*$ and *n* recalls the central idea of the Jury Theorem. The expected value grows with *n* because there is information aggregation (although imperfect, as we have seen in the five member case). In the present paper we do not address the issue of the optimal committee size. We take the size as exogenous, as it is likely to be determined on the ground of other criteria than the optimality of the voting behavior of the committee.¹⁴ Moreover, the optimal size of the committee can be bounded by information costs (e. g., see Persico, 2004). When information acquisition is costly, each member has an incentive to acquire information only insofar as his benefit, which depends on the probability of being pivotal, is not lower than his cost. As in our equilibrium the probability of being pivotal decreases in size, this may set a limit to the optimal size of the committee.¹⁵

 $^{^{14}}$ For example, the need to represent different stake olders or to balance different powers or just plain political criteria.

¹⁵We do not develop this aspect in the present paper because it would distort the attention away from our main focus. Costly information acquisition can be studied along the lines set in Persico (2004) who analyzes the optimal voting rule (in terms of both incentives to acquire information and information aggregation) in a similar setting. See also Harris and Raviv (2008) on the effects of costly information acquisition on the optimal size of boards of directors.

5 Heterogeneous preferences

Having determined the optimal equilibria, we compare this benchmark to the outcome of a committee composed of members with heterogeneous preferences. Consider again a committee with five members (n = 2), in the case where $\alpha = \frac{1}{2}$, but let now members be either of type M or B. If B members hold the majority, the case is trivial because the committee always approves the project and the M members are never pivotal. Then, we concentrate on the remaining two interesting cases in which the committee is composed of : i) four value-maximizing members M_i , i = 1, 2, 3, 4, and one biased members B_5 ; and ii) three value-maximizing members M_i , i = 1, 2, 3, and two biased members B_j , j = 4, 5.

We start from the latter case and we show that there exists a unique equilibrium. Given that B members always vote "yes", independently of their information, M members vote according to their information whenever informed, and vote "no" when they are uninformed. Such rejection is optimal, because the probability that an uninformed member is pivotal is higher when the state of the world is L. In this case, we can easily compute:

$$E(v)_{M=B+1} = \frac{1}{2} [Y(\cdot|v=1) - Y(\cdot|v=-1)] = \frac{1}{2} \left[\frac{7}{8} - 0\right] = \frac{7}{16}$$

Quite surprisingly, the performance of this committee is the same as the optimal performance of the committee composed of unbiased members only. In addition, this equilibrium is unique. Thus, we can say that the heterogeneous committee ensures the optimal outcome, provided that the M members outnumber B members by just one vote.

Consider now the case where the committee is composed of M_i , i = 1, 2, 3, 4, and B_5 . Recall that the dominant strategy of an informed M member is to vote according to his information, and that of the unique B member is to always approve the project. Then, three types of equilibria emerge, differing as to the behavior of uninformed M members. In the equilibria of the first type, two of the M members compensate for each other in pure strategies when uninformed, and the other two M members vote "no". In the equilibrium of the second type, all of the four M members play the same mixed strategy, when uninformed. In the equilibria of the third type, one of the M members votes "no", while the other three M members play the same mixed strategy, when uninformed.

While the equilibrium of the first type is optimal, yielding expected value

$$E(v)_{M>B+1}^{PS} = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] = \frac{1}{2}\left[\frac{7}{8} - 0\right] = \frac{7}{16}$$

the equilibrium of the second type is suboptimal. We find in fact the following solution:

 $m_1 = m_2 = m_3 = m_4 = \frac{5-\sqrt{13}}{6}$, and

$$E(v)_{M>B+1}^{MS} = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] = .384973 < \frac{7}{16}$$

Intuitively, when the M members outnumber the B members by more than one, the former have different choices. It may be the case that one single member offsets the bias of B_5 in pure strategies and the remaining members compensate for each other in pure strategies, or that all of the M members play the same mixed strategy with the aim to collectively contrast the bias of B_5 . This is the reason why the symmetric mixed strategy of the M members in the second type of equilibrium is now biased towards rejection, $m_i < 1/2$, i = 1, 2, 3, 4.

Moreover, another type of equilibrium is possible where the M members collectively contrast the bias of B_5 : one biased member always votes "yes", one uninformed unbiased member plays a pure strategy ($m_1 = 0$), and the remaining members, whenever uninformed, play a symmetric mixed strategy ($m_2 = m_3 = m_4 = \frac{1}{3}$). Such an equilibrium yields:

$$E(v)_{M>B+1}^{PMS} = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] = .388889.$$

Hence:

$$E(v)_{M>B+1}^{MS} < E(v)_{M>B+1}^{PMS} < E(v)_{M>B+1}^{PS}$$

With more than 5 members, there may arise additional equilibria, analogous to the fourth type of equilibrium in Proposition 1. Such equilibria however are shown to be suboptimal in Proposition 2, which generalizes our results.

Proposition 2 Consider a committee with 2n + 1 members where members of type B always approve the project and informed members of type M always vote according to their information. We can distinguish two cases:

i) if there are n members of type B and n+1 members of type M the game has a unique equilibrium in which all the members of type M always vote "no" when uninformed. This unique equilibrium is optimal;

ii) if there are n - k members of type B (n > k > 0) and n + 1 + k members of type M, the voting game may have multiple equilibria. There always exists optimal equilibria where 2kmembers of type M compensate for each other in pure strategies and the remaining n - k + 1members of type M vote "no" when uninformed; other equilibria are suboptimal.

From the above proposition, Corollary 2 immediately follows.

Corollary 2 The expected value $E(v)^*$ is not increased by increasing the proportion of valuemaximizing members above $\frac{n+1}{2n+1}$. For a given size of the committee, increasing the proportion of the M members is not profitable, provided they already hold the majority. By increasing the proportion of the M members, optimal equilibria can still be obtained but there may also exist other kind of equilibria. Thus, if value-maximizing members outnumber biased members by only one vote, the situation is greatly simplified with respect to our benchmark case because the optimal equilibrium is unique. Hence, we refer to this composition as optimal and to such a committee as the optimal heterogeneous committee.

We know from Corollary 1 that increasing the size of the committee (increasing n), increases $E(v)^*$, when the optimal equilibrium is played. Since an increase in n may now result in an increase in the number of the B members, this point deserves some attention. Consider the case with n members of type B and n + 1 members of type M and recall that

$$E(v) = \frac{1}{2} [Y(\cdot | v = 1) - Y(\cdot | v = -1)].$$

The probability of approving the project when the state of nature is unfavorable, $Y(\cdot|v = -1)$, is still equal to zero after an increase in n, because the effect of additional biased members is compensated for by the additional M members voting "no" when uninformed. On the contrary, the probability of approving the project when it is profitable, $Y(\cdot|v = 1)$, increases with n, because it is equal to the probability that at least one of the n + 1 members of type M is informed. Hence adding new members (including biased ones) is profitable.¹⁶

6 Discussion

We have assumed that the members of the committee cannot abstain and cannot communicate. We now briefly comment on these two assumptions.

Abstention

What happens if we relax our no-abstention assumption? First of all, note that allowing abstention would bring some new issues and restrictions into the picture. For instance, it should be arbitrarily stated what happens when everyone abstains, and an ad hoc rule (i.e., a probability to implement the project) should be applied. Moreover, it is unclear whether to look at simple majority of members or simple majority of actual votes.

In general, however, abstention would improve the performance of a committee composed only of unbiased members (our benchmark). With simple majority of actual votes, there clearly is an equilibrium in which uninformed members abstain. Notice that any ad hoc rule for the case in which all members abstain can induce such a behavior. In fact, if

¹⁶Clearly our comment on the optimal size of the committee following Corollary 1 still applies.

at least one member is informed, the correct decision is made with certainty. In other words, the wrong decision is made with probability $\frac{1}{2}$ only when nobody is informed. If the committee is composed of 2n + 1 unbiased members, in this equilibrium the expected value is $E(v) = \frac{1}{2}[1 - (1 - \alpha)^{2n+1}] > E(v)^*$.

Things are more complicated if there are k biased members, with $1 \le k < n$. In this case, k unbiased members should not abstain when uninformed but should vote "no" in order to contrast the biased members' votes¹⁷. Moreover, as it may occur that k unbiased members vote "no", while the other M members abstain, the tie-breaking rule should prescribe not to implement the project so as to take care of the case where the only informed M members are among those who (when uninformed) contrast the B members.

Notice however that the no-abstension assumption implies no loss of generality in the case of our optimal heterogeneous committee (n + 1 unbiased members and n biased members). Indeed, suppose abstention is allowed: biased members always vote "yes" and any unbiased member still finds it optimal to condition his strategy on being pivotal. The reason is the following. Suppose in a symmetric equilibrium all the other unbiased members abstain when uninformed. If the remaining one is pivotal, this means that the other unbiased members voted "no", hence voting "no" is optimal. This is true for all unbiased members, hence "abstention" is not an equilibrium strategy. This is true even in all other asymmetric (putative) equilibria where some unbiased members vote "no" when uninformed and some others abstain. The most extreme case obtains when all the other unbiased members vote "no" when uninformed. Still, the probability that at least one of the other members voted "no" because he is informed and the true state in -1 is higher than the probability that they all voted "no" because they are uninformed (the only case where the remaining member would be indifferent among voting "yes", "no" or abstaining). Hence, voting "no" instead of abstaining is optimal (formula and calculation are the same as the ones in the proof of Proposition 2).

Communication

If we allow members to communicate prior to voting, unbiased members will certainly communicate any information they possess because information aggregation among them may improve the outcome. Indeed, it can be easily verified that perfect information aggregation among M members is feasible. It suffices that a single M member is informed to reach the correct decision. Coherently with the literature (see, e.g. Gerling et al., 2005), communication can be introduced as a pre-voting stage where members of type M simultaneously send costless messages about their information sets. Recall that the information set of a generic member i is $\Omega_i = \{\omega_i\}$, with $\omega_i \in \{H; L\}$, when i is informed, and $\Omega_i = \{H, L\}$

¹⁷For simplicity, we only consider pure strategies for the M members.

when *i* is uninformed. Consider the case where member *i* can send a message $\sigma_i \in \{H, L, 0\}$, where $\sigma_i = 0$ means that *i* sends no information. Messages update *M* members' information sets.¹⁸ Moreover, assume that an informed member, whenever indifferent, sends a truthful message. Then, equilibrium strategies prescribe that each informed *M* member sends a truthful message and votes according to his information. Furthermore, each uninformed *M* member receiving at least one $\sigma_i = L$ votes according to the received message(s), and indifferently votes either "yes" or "no" otherwise. This implies that in equilibrium full aggregation of information among *M* members is possible. As a result, it is optimal to increase the number of unbiased members as much as possible, because the outcome now depends on their (absolute) number and not on their proportion to biased members as in the optimal committee of the no-communication case. If the committee is composed of 2n + 1 unbiased members, the expected value becomes $E(v) = \frac{1}{2}[1 - (1 - \alpha)^{2n+1}]$ because the wrong decision is made with probability 1/2 only when nobody is informed, and this occurs with probability $(1 - \alpha)^{2n+1}$.

However, if we consider the optimal committee of the no communication case with n + 1 members of type M and n members of type B, the introduction of communication cannot improve on the outcome $E(v)^*$. In such a committee, the right decision is made with probability 1 when at least one out of the n + 1 M members is informed, and with probability $\frac{1}{2}$ when no M member is informed. But this is precisely what happens in the case without communication, and consequently the expected value of the project reaches the same level $E(v)^* = \frac{1}{2}[1-(1-\alpha)^{n+1}]$. Indeed, the voting strategies of the M members in the case with no communication (contrasting biased members and leaving the decision to possibly informed members) minimize the information required to reach the best possible outcome, $E(v)^*$.¹⁹ We can then conclude that, for a committee composition with n + 1 members of type M and n members of type B, communication cannot improve the voting outcome.

¹⁸Alternatively we can assume that messages are exchanged among all the members and enter everybody's information set. Notice however that biased members cannot commit to send truthful messages because of their strong bias. Thus, they would never be believed. This is equivalent to assuming that they do not send any message, i.e. $\sigma_B = 0$. On the other hand members of type B, given their preferences, would not change their strategies even if they received a message revealing that the state of nature is L. For these reasons we focus on the message strategies of the M members.

¹⁹The introduction of communication results in the expansion of the set of equilibria. When no information is revealed, M members now know that nobody is actually informed and thus have no strategic reason to contrast biased members and make other unbiased members pivotal. They can indifferently cast any vote and thus multiple equilibria arise: there now also exist equilibria where some unbiased members vote "no" after observing $\sigma_i = H$. These additional equilibria may entail an unconvincing behavior on the part of the M members, nonetheless they all yield $E(v)^* = \frac{1}{2}[1 - (1 - \alpha)^{n+1}]$. Notice however that, contrary to what happens in the type ii) equilibria of proposition 1, this multiplicity does not entail any coordination problem: whatever the choice of the M members receiving message $\sigma_i = H$, an equilibrium with expected value $E(v)^*$ is reached.

7 Conclusions

We have analyzed the voting behavior of a small committee that has to approve or reject a project whose return is uncertain. Members have heterogenous preferences: some members want to maximize the expected value while others have a bias towards project approval and disregard their private information. More precisely, we have shown that, in the absence of communication among members, heterogeneous committees can function at least as well as committees with homogeneous value-maximizing members. In particular, when value maximizers outnumber biased members by just one vote, the presence of biased members can improve the voting outcome by simplifying the strategies of the value maximizers: the equilibrium becomes unique and yields the optimal outcome. For a given committee size, increasing the number of value-maximizing members above the minimum that ensures majority does not increase the expected value and gives rise to additional suboptimal equilibria.

Despite being quite simple, we believe our framework can be easily applied to explain voting behaviors in a number of different small decisional bodies such as monetary policy committees, juries, boards of directors, and so on. In all of these committees, it is not uncommon to observe dissent voting. We explain such dissent as the result of optimal voting strategies, given an optimal composition rule of the committee itself. Furthermore, our result shows that the composition actually used in some small committees (for instance, in the Bank of England Monetary Policy Committee or the Italian Constitutional Court) is optimal if members are diverse and communication is limited.

8 Appendix

8.1 Proof of Proposition 1

Recall that value-maximizing members choose their strategies conditioning on being pivotal. Then, each informed member votes according to his information, as this maximizes the probability of making the correct decision. Thus, in what follows we only focus on the voting strategies of uninformed members.

Considering a committee composed of 2n + 1 members of type M, we prove that there only exist i) multiple equilibria where 2n individuals compensate in pure strategies, ii) a unique equilibrium where all members compensate in mixed strategies playing $m_j = 1/2$, and multiple equilibria where 2n - 2k, k = 1, 2...n - 1, individuals compensate in pure strategies and 2k + 1 individuals play $m_k = 1/2$, iii) multiple equilibria where $(n - k_1)$ members choose $m_j = 0$ ($m_z = 1$) and ($n - k_2$) members choose $m_z = 1$ ($m_j = 0$) with $k_1 < k_2 \leq n$ while all the others, $k \neq j, z$, choose the same mixed strategy $m_k < \frac{1}{2}$ ($m_k > \frac{1}{2}$). Finally, we prove that equilibria of type i) are optimal in that they maximize E(v).

i) There exist multiple equilibria where n members choose $m_j = 1$, n members choose $m_z = 0$, and one member, denoted by $M_i \ i \neq j, z$, chooses $m_i \in [0, 1]$. There cannot exist an equilibrium where more than n+1 members choose either $m_j = 1$ or $m_z = 0$.

We prove the existence of these equilibria in four steps. First we prove that player i is voting optimally, given the strategies of the other 2n players; then we prove that the other 2n members are voting optimally as well (steps 2 and 3). Finally, we prove that these are the only equilibria where at least n + 1 members choose either $m_j = 1$ or $m_z = 0$.

1. If n members choose $m_j = 1$, n members choose $m_z = 0$, the best response of M_i , $i \neq j, z$, is to choose $m_i \in [0, 1]$.

When n members are voting "yes" and n members are voting "no" M_i is pivotal in both states of the world with the same probability. Indeed, when v = 1, M_i is pivotal when everybody else is uninformed or when the only informed members are those n members who would vote "yes" even if uninformed (thus not changing their votes whether informed or not). As the former case (everybody is uninformed) can be considered as a sub case of the latter, the probability that M_i is pivotal is

$$(1-\alpha)^n \left[\sum_{j=0}^n \frac{n!}{j!(n-j)!} \alpha^{n-j} (1-\alpha)^j \right] = (1-\alpha)^n.$$

where $\frac{n!}{j!(n-j)!}$ represents the number of combination with n-j informed members among the n members who vote "yes" if uninformed, and the term in brackets is equal to 1 from the binomial theorem. When v = -1, M_i is pivotal when everybody else is uninformed or when the only informed members are those who would vote "no" even if uninformed. Then the probability that M_i is pivotal is again $(1-\alpha)^n$. Hence, M_i is indifferent between the possible values of $m_i \in [0, 1]$.

2. If n members choose $m_j = 1$, n - 1 members choose $m_z = 0$, and member M_i , $i \neq j, z$, chooses $m_i = 0$ the best response of the remaining member (denoted by $k, k \neq i, j, z$) is to choose $m_k \in [0, 1]$; if M_i , chooses $m_i > 0$ the best response of M_k is to choose $m_k = 0$.

If M_i chooses $m_i = 0$, we are back to point 1. So the optimal response of the remaining M_k is $m_k \in [0, 1]$. If instead M_i chooses $m_i = 1$, then M_k is pivotal only when v = -1. Consequently, M_k chooses $m_k = 0$. But M_k chooses $m_k = 0$ even if $1 > m_i > 0$ because M_k is pivotal with a higher probability in v = -1 than in v = 1.

3. If n-1 members choose $m_j = 1$, n members choose $m_z = 0$, and member M_i , $i \neq j, z$, chooses $m_i = 1$ the best response of the remaining member (denoted by $k, k \neq i, j, z$) is to choose $m_k \in [0, 1]$; if M_i , chooses $m_i < 1$ the best response of M_k is to choose $m_k = 1$.

The argument is symmetric to the one used at point 2.

Finally, note that any member can be in the position of M_i , or in that of an M_j voting "yes", or also in that of an M_z voting "no" or randomizing when uninformed. Thus, there is a multiplicity of equilibria such as the one we are considering.

4. There cannot exist an equilibrium where more than n + 1 members choose either $m_j = 1$ or $m_z = 0$.

Consider what happens if more than n + 1 members vote "yes", i.e. suppose n + 1 + kmembers $(k \in \{1, 2, 3, ..., n - 1\})$ choose $m_j = 1$. Then every remaining member knows that he is pivotal with a higher probability when v = -1. Hence, the remaining n - k members choose $m_z = 0$. However, this cannot be an equilibrium. Also members voting $m_j = 1$ know that they are pivotal with a higher probability when v = -1. Hence, as long as more than n + 1 members still vote "yes" when uninformed $(k \neq 0)$, they have an incentive to change their strategy and vote "no" when uninformed.

A symmetric argument can be used to analyze what happens if more than n+1 members vote "no" when uninformed, i.e. if n+1+k members ($k \in \{1, 2, 3, ..., n-1\}$) choose $m_z = 0$ and consequently to rule out the existence of such equilibria.

ii) There exist multiple equilibria where n - k members choose $m_j = 1, n - k$ members choose $m_z = 0$, and 2k + 1 members choose $m_k = 1/2$ for k = 1, 2..n - 1. If 2k + 1 choose $m_k = 1/2$ for k = 1, 2..n - 1 for an equilibrium to exist, members choosing pure strategies must compensate each other. If 2(n - k) members compensate in pure strategies, there cannot exist an equilibrium with at least one member choosing $m_k \neq 1/2$. There exists a unique equilibrium where all the 2n + 1 members play mixed strategies (k = n). Such an equilibrium is symmetric with $m_k = \frac{1}{2}$.

We prove the existence of these equilibria in three steps.

1. If n-k members choose $m_j = 1$, n-k members choose $m_z = 0$, and 2k members choose $m_k = 1/2$, k = 1, 2...n, the best response of the remaining member (denoted by $i, i \neq j, k, z$) is to choose $m_i = \frac{1}{2}$.

Both when v = 1 and when v = -1, M_i is pivotal if a) everybody is uninformed and n members vote "yes" while the other n members vote "no", or b) no more than n members are informed and vote accordingly, while uninformed members vote in such a way that results in n members voting "yes" and n members voting "no". Given that the other members either compensate in pure strategies or choose $m_k = \frac{1}{2}$, M_i is pivotal with the same probability in both states of the world. Since both states are equally possible, M_i is then indifferent among any $m_i \in [0, 1]$.

This holds true for every member playing a mixed strategy. If nobody plays a pure

strategy (k = n) it immediately follows that $m_k = \frac{1}{2}$ for j = 1, ..., 2n + 1, sustains an equilibrium of the game.

2. If there are more members choosing $m_j = 1$ ($m_z = 0$) than members choosing $m_z = 0$ ($m_j = 1$), while the other members but one choose $m_k = \frac{1}{2}$, the remaining member has a pure strategy as his best response. If 2k + 1 members choose $m_k = \frac{1}{2}$, the other members must compensate each other in pure strategies.

If there are more members choosing $m_j = 1$ $(m_z = 0)$ than members choosing $m_z = 0$ $(m_j = 1)$ while all the others but member M_i , $i \neq j, z, k$, choose $m_k = \frac{1}{2}$, the best response of M_i is to choose $m_i = 0$ $(m_i = 1)$ because M_i is pivotal with a higher probability when v = -1 (v = 1) than when v = 1 (v = -1).

From point 1. and 2., it follows that a situation where n - k members choose $m_j = 1$, n - k members choose $m_z = 0$, and 2k + 1 members choose $m_k = 1/2$, k = 1, 2..n - 1, constitutes an equilibrium of the game. Moreover it follows from 2., that if 2k + 1 choose $m_k = 1/2$, k = 1, 2..n - 1, for an equilibrium to exist, members choosing pure strategies must compensate each other. Given that k can take values 1, 2..n - 1 and that any member can be in the position of a j, a z or a k member, there exists a multiplicity of such equilibria.

3. There cannot exist either an equilibrium in mixed strategies with $m_k \neq \frac{1}{2}$ for one or more members or an equilibrium with at least one member choosing $m_k \neq 1/2$ when n-k members choose $m_j = 1$, n-k members choose $m_z = 0$ for k = 1, 2...n - 1 and the other 2k members choose mixed strategies.

If M_i were to choose $m_i > \frac{1}{2}$ $(m_i < \frac{1}{2})$ while 2n - 1 members choose $m_k = \frac{1}{2}$, the best response of the remaining member denoted by $j, j \neq i, k$, would be $m_j = 0$ $(m_j = 1)$, because M_j would be pivotal with a higher probability when v = -1 (v = 1) than when v = 1 (v = -1). With 2n - 1 members choosing $m_k = \frac{1}{2}$ and M_j choosing $m_j = 0$ $(m_j = 1)$, however the best response of M_i becomes $m_i = 1$ $(m_i = 0)$, because M_i would be pivotal with a higher probability in v = 1 (v = -1) than in v = -1 (v = 1).

A similar argument holds true if member M_i , choosing $m_i > \frac{1}{2}$, were compensated by another member, denoted by h, choosing $m_h < \frac{1}{2}$ and such that $m_i + m_h = 1$. In this case no other member has an incentive to deviate from $m_j = \frac{1}{2}$, but it immediately appears that $m_h < \frac{1}{2}$ is not a best response. Given that M_h is pivotal with a higher probability in v = -1than in v = 1, his best response is $m_h = 0$.

More generally, by applying the same line of reasoning, it can be verified that there cannot exist an equilibrium with $m_i \neq \frac{1}{2}$ for at least one *i*, because as soon as one or more agents choose $m_i \neq \frac{1}{2}$, there is at least one agent (possibly one of those choosing $m_i \neq \frac{1}{2}$) who has a pure strategy as his best response. Hence the only equilibrium in mixed strategies is the one with $m_i = \frac{1}{2}$ for i = 1, 2...2n + 1. Exactly the same argument can be applied in order to rule out equilibria where 2k + 1 members choose mixed strategies $m_k \neq \frac{1}{2}$, when there are 2(n-k) members, k = 1, 2..n - 1, compensating each other in pure strategies.

iii) There may exist multiple equilibria where $(\mathbf{n} - \mathbf{k}_1)$ members choose $m_j = 0$ $(m_z = 1)$ and $(\mathbf{n} - \mathbf{k}_2)$ members choose $m_z = 1$ $(m_j = 0)$ with $\mathbf{k}_1 < \mathbf{k}_2 \le n$ while all the others, $k \ne j, z$, choose the mixed strategy $m_k > \frac{1}{2}$ $(m_k < \frac{1}{2})$.

We do not need to characterize all possible equilibria of this type. To our purpose, it is sufficient to show that there may exist equilibria where some members use a mixed strategy $m \in (0,1)$ and the other members choose pure strategies. More precisely, we build an example for the case of n = 3 and $k_1 = 1 < k_2 = 2$. This is the smallest board where some members may compensate in pure strategies, while another member chooses a pure strategy and the rest choose mixed strategies (if n = 2 there could be an equilibrium with one member choosing a pure strategy while the rest are choosing the same mixed strategy).

First of all by the argument used at point ii)1, for this to be an equilibrium, members choosing mixed strategies must choose the same value of m. Hence, consider the case where $m_1 = m_2 = 0, m_3 = 1, m_4 = m_5 = m_6 = m_7 = \overline{m}$. The value of \overline{m} can be derived by maximizing the function V(m) defined as follows for $m \in [0, 1]$

$$V(m) = E(v) = \frac{1}{2} [Y(\cdot | v = 1) - Y(\cdot | v = -1)] when$$

$$m_1 = m_2 = 0$$

$$m_3 = 1$$

$$m_4 = m_5 = m_6 = m_7 = \overline{m}.$$

Before showing that the solution to the maximization of V(m) identifies the equilibrium mixed strategy, we show that such maximization has an interior solution. Given that (a) all M players have identical payoffs equal to E(v) and (b) the game is symmetric with respect to the M players, neither m = 0 nor m = 1 maximize V(m). To check this, suppose that V(m)is maximized by m = 1 and consider M_4 . Given $m_5 = m_6 = m_7 = m = 1$, M_4 is more likely to be pivotal in v = -1 than in v = 1, then his best reply is not $m_4 = 1$ but it is $m_4 = 0$, because with such a choice he can obtain the highest possible value of E(v). Since a player's expected payoff is linear in his own mixed strategy, we can then replace $m_4 = 1$ with $m'_4 = 0 + \varepsilon$ for any $\varepsilon \in (0, 1)$ and raise E(v). Given (a) and (b), this is true also for M_5 , M_6 and M_7 . Since E(v)is a polynomial in $(m_1, m_2, ..., m_7)$, first-order effects dominate for sufficiently small $\varepsilon > 0$, if m = 1 is replaced by $m' = 0 + \varepsilon$ in V(m) so that $V(0 + \varepsilon) > V(1)$, contradicting that V(m) is maximized by m = 1. An analogous argument rules out that V(m) is maximized by m = 0. Since V(m) is continuos in m, there then exists a value $\overline{m} \in (0, 1)$ that maximizes V(m), and, considering that if it were $\overline{m} \leq 1/2$ the above argument (on possible increases of E(v) by changing m) could be applied again, we can conclude that $\overline{m} > 1/2$.

In order to show that $m_1 = m_2 = 0$, $m_3 = 1$, $m_4 = m_5 = m_6 = m_7 = \overline{m}$ is actually an equilibrium, consider first of all one of the members choosing \overline{m} , e.g. M_4 . For this to be an equilibrium, \overline{m} must represent the best reply of M_4 . Suppose instead that M_4 can raise his payoff, E(v), by choosing $m'_4 \neq \overline{m}$ and let $m'_4 = \overline{m} + \Delta$. Since a member's payoff is linear in his mixed strategy, we can replace \overline{m} with $\overline{m} + \varepsilon \Delta$ for any $\varepsilon \in (0, 1)$ and so raise E(v). Then the above argument can be repeated to show that for sufficiently low ε , it is possible to replace \overline{m} with $\overline{m} + \varepsilon \Delta$ for M_4 , M_5 , M_6 , M_7 and obtain $V(\overline{m} + \varepsilon \Delta) > V(\overline{m})$, thus contradicting that \overline{m} maximizes V(m).

To check that also M_1 , M_2 , and M_3 are choosing their best replies, consider one of the members choosing $m_i = 0$, e.g. M_1 . As $\overline{m} > 1/2$, the probability that M_1 is pivotal is higher in v = -1 than in v = 1, consequently his best reply is to choose $m_1 = 0$. The same clearly holds for M_2 . Consider instead M_3 . Given $m_1 = m_2 = 0$ and the fact that \overline{m} is calculated by maximizing E(v) in a case where only one member is choosing m = 1, M_3 is pivotal with a higher probability in v = 1 than in v = -1. Consequently his best response is to actually choose $m_3 = 1$. Since any of the members can be in the position of M_1 , M_2 , M_3 , M_4 , M_5 , M_6 , M_7 , there exist a multiplicity of such equilibria.

By solving this problem for $\alpha = \frac{1}{2}$ we find:

$$m_4 = m_5 = m_6 = m_7 = 0.656537;$$

 $E(v) = 0.448531$

Clearly an analogous equilibrium can be constructed where $m_1 = m_2 = 1$, $m_3 = 0$, $m_4 = m_5 = m_6 = m_7 = m_k < 1/2$.

iv) There cannot exist other equilibria than those considered at points i-iii).

From points i)-iii) it immediately follows that all possible combinations of strategies have been considered.

v) The equilibria with compensation in pure strategies obtained at point i) maximize E(v)

In order to compare the expected value obtained in different equilibria, consider that E(v) can be written as

$$E(v) = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)]$$

= $\frac{1}{2}[1 - Y(\cdot|v=-1) - (1 - Y(\cdot|v=1))]$

where $Y(\cdot | v = -1) + (1 - Y(\cdot | v = 1))$ is the probability of making the wrong decision.

Consider the equilibrium where every member plays the mixed strategy $m_k = 1/2$. We know from point ii) 1. that when there are 2n members choosing $m_k = 1/2$, the 2n + 1thmember, denoted by M_i , is in fact indifferent among any $m_i \in [0, 1]$. This means that he cannot raise E(v) by playing one instead of another strategy because as $Y(\cdot | v = -1)$ is lowered (raised) by choosing a specific strategy, $(1 - Y(\cdot | v = 1) \text{ is raised (lowered) by exactly})$ the same amount. Then the expected value is the same for any $m_i \in [0,1]$. This implies that if M_i were to choose $m_i = 1$, E(v) would stay at $E(v)^{MS}$, even if this would not be an equilibrium situation. Consider another member, denoted by M_h . From point ii)2 we know that in such non equilibrium situation, his best response is to choose $m_h = 0$, because he is pivotal with a higher probability in v = -1 than in v = 1 and by choosing $m_h = 0$, he can raise E(v) above $E(v)^{MS}$. We know from point ii) that the situation where M_i chooses $m_i = 1$, M_h chooses $m_h = 0$ and all the other members choose $m_k = 1/2$ constitutes an equilibrium. Then, such an equilibrium yields $E(v) > E(v)^{MS}$. The above argument, however, can be applied again by singling out one of the members choosing $m_k = 1/2$ who is in fact indifferent among any $m \in [0, 1]$, and letting him choose m = 1 (m = 0). The new situation will not constitute an equilibrium but, there will again be another member who can raise E(v) by choosing m = 0 (m = 1), and this new situation will constitute an equilibrium. Clearly the argument can be recursively repeated to the point where an equilibrium in which 2n members compensate in pure strategies while the remaining member chooses $m_k \in [0, 1]$, is reached. Consequently, an equilibrium with compensation in pure strategies yields a higher E(v) than the equilibrium in mixed strategies, as well as than the equilibria with 2(n-k)agents compensating in pure strategies and k + 1 agents choosing $m_k = 1/2$.

Consider now the case where $(n - k_1)$ members choose $m_j = 0$ and $(n - k_2)$ members choose $m_z = 1$ with $k_1 < k_2 \le n$, while all the others, $k \ne j, z$, choose the mixed strategy $m_k > \frac{1}{2}$. In particular, consider the equilibrium characterized at point iii. The same argument as above can be applied with slight modifications.

Consider M_4 who is choosing the mixed strategy m_k . M_4 is in fact indifferent among any $m_4 \in [0, 1]$ meaning that E(v) is not modified if he changes his strategy (provided that the others do not change theirs). So let him switch to $m_4 = 1$. E(v) is still 0.448531 but this is no longer an equilibrium because $m_k = 0.656537$ was calculated so as to make any of M_4 , M_5 , M_6 , M_7 indifferent among $m_k \in [0, 1]$ when the four of them where playing m_k , but now there are only three of them choosing m_k implying that each is pivotal with a higher probability in v = -1 than in v = 1. Consider then M_5 . His best response is to choose $m_5 = 0$, meaning that, by doing so, he can raise E(v) above 0.448531. This is not an equilibrium, because if $m_1 = m_2 = m_5 = 0$ and $m_3 = m_4 = 1$ and $m_7 = m_k$, the best response of M_6 is to

choose $m_6 = 1$ as he is pivotal with a higher probability in v = 1 than in v = -1. But this means that, by doing so, M_6 can further raise E(v). Moreover as now $m_1 = m_2 = m_5 = 0$ and $m_3 = m_4 = m_6 = 1$, an equilibrium with compensation in pure strategies has been established.

This argument can clearly be generalized to any equilibrium where $(n - k_1)$ members choose $m_j = 0$ $(m_j = 1)$ and $(n - k_2)$ members choose $m_z = 1$ $(m_z = 0)$ with $k_1 < k_2 \le n$, while all the others, $k \ne j, z$, choose the mixed strategy $m_k > \frac{1}{2}$. It is sufficient to single out one of the members choosing the mixed strategy and let him switch to a pure strategy. Then, by having the the others successively switch to their best responses, the equilibrium with compensation in pure strategies is reached as an improvement upon the starting point.

8.2 Proof of Corollary 1

The Corollary follows immediately considering that

$$\frac{\partial [E(v)^*]}{\partial \alpha} = \frac{n+1}{2} (1-\alpha)^n > 0, \qquad \frac{\partial [E(v)^*]^2}{\partial \alpha^2} = -\frac{n(n+1)}{2} (1-\alpha)^{n-1};$$

and that the marginal expected value when n increases is

$$\Delta_{2n+1} E(v)^{PS} \equiv \frac{1}{2} [1 - (1 - \alpha)^{(n+1)+1}] - \frac{1}{2} [1 - (1 - \alpha)^{n+1}] = \frac{\alpha}{2} (1 - \alpha)^{n+1}$$

which is clearly decreasing in n.

8.3 Proof of Proposition 2

Recall that value-maximizing members choose their strategies conditioning on being pivotal. Then, each informed M member votes according to his information, as this maximizes the probability of making the correct decision. Thus, in what follows we only focus on the voting strategies of uninformed members. The proof is organized as follows. We prove that:

i) if the committee is composed of n+1 value-maximizing members and n biased members, there exists a unique equilibrium where each M member votes "no" when uninformed;

ii₁) if the committee is composed of n + 1 + k value-maximizing members and n - k biased members, there always exist equilibria where n - k value-maximizing members vote "no" when uninformed and the remaining 2k value-maximizing members compensate for each other in pure strategies.

ii₂) if the committee is composed of n+1+k value-maximizing members and n-k biased members (n > k > 0), there may exist equilibria where some or all the value-maximizing

members play the same mixed strategy;

iii) the equilibria sub i) and sub ii₁) are optimal while the equilibria sub i₂ are suboptimal. i) In a committee with n + 1 value-maximizing members and n biased members there exists a unique equilibrium where all the M members vote "no" whenever uninformed (that is, $m_i = 0; i = 1, 2, ..., n + 1$).

Consider member M_{n+1} . When v = 1, M_{n+1} is pivotal only if all the other M members are uninformed and vote "no", which happens with probability:

$$(1-\alpha)^n \prod_{i=1}^n (1-m_i)$$

When v = -1, M_{n+1} is pivotal if:

- all the other M members are uninformed and vote "no", which happens with probability

$$(1-\alpha)^n \prod_{i=1}^n (1-m_i),$$

- all the other M members are informed, which happens with probability

 α^n ,

- at least one (but not all) of the other M members is informed and the others vote "no" when uninformed, which happens with probability

$$\sum_{h=1}^{n} \frac{n!}{h!(n-h)!} \alpha^{n-h} (1-\alpha)^h \prod_{i=1}^{h} (1-m_i).$$

where $\frac{n!}{h!(n-h)!}$ represents the number of combination with h uninformed value-maximizing members and n-h informed value-maximizing members. It is straightforward that M_{n+1} is pivotal with a higher probability when v = -1. Hence M_{n+1} chooses $m_{n+1} = 0$. As the same reasoning holds for any other value-maximizing member $i \neq n+1$, it follows that every Mmember will vote "no" when uninformed.

Finally, note that we have not restricted m_i , $i \neq n + 1$, to any particular value, so the result also proves that this equilibrium is unique.

ii₁) In the case of n-k biased members (n > k > 0) and n+1+k value-maximizing members, there exist multiple equilibria with n-k+1 value-maximizing members voting against the project and 2k value-maximizing members compensating for each other in pure strategies.

We prove the existence of these equilibria in three steps. In the first step, we prove that when n - k value-maximizing members vote against the project and 2k value-maximizing members compensate for each other, the remaining M member has still an incentive to vote against the project; in the second step, we prove that when n - k value-maximizing members vote against the project to contrast the n - k biased members, and a majority of the other value-maximizing members also vote against the project, the remaining M member has an incentive to compensate, voting "yes". Finally, we show that there are no other equilibria in pure strategies.

1. If n value-maximizing members choose $m_z = 0$, and k value-maximizing members choose $m_j = 1$, the best response of M_i , $i \neq j, z$, is to choose $m_i = 0$.

When v = 1, M_i is pivotal if all the value-maximizing members are uninformed or if at least one of those k value-maximizing members who choose $m_j = 1$ when uninformed, is in fact informed. Thus, M_i is pivotal with probability

$$(1-\alpha)^n \left[\sum_{j=0}^k \frac{k!}{j!(k-j)!} \alpha^{k-j} (1-\alpha)^j \right] = (1-\alpha)^n$$

where $\frac{k!}{j!(k-j)!}$ represents the number of combination with j uninformed value-maximizing members, k - j informed M members and the term in brackets is equal to 1 from the binomial theorem. When v = -1, M_i is pivotal if all the M members are uninformed or if at least one of those n value-maximizing members who choose $m_z = 0$ when uninformed, is in fact informed. Then M_i is pivotal with probability

$$(1-\alpha)^k \left[\sum_{z=0}^n \frac{n!}{z!(n-z)!} \alpha^{n-z} (1-\alpha)^z \right] = (1-\alpha)^k$$

Given that $(1 - \alpha)^k > (1 - \alpha)^n$, the probability that M_i is pivotal is higher when v = -1 than when v = 1. Hence M_i chooses $m_i = 0$.

2. If n+1 value-maximizing members choose $m_z = 0$ and k-1 value-maximizing members choose $m_j = 1$, the best response of M_i , $i \neq j, z$ is to choose $m_i = 1$

When v = 1, M_i is pivotal if only one of the n + 1 value-maximizing members choosing $m_z = 0$ is informed and votes "yes". This happens with probability

$$(n+1)(1-\alpha)^n\alpha.$$

On the contrary, M_i is never pivotal when v = -1. Hence, he chooses $m_i = 1$.

Finally, note that any M member can be in the position of M_i or in that of an M_j voting

"yes", or also in that of an M_z voting "no". Thus, there is a multiplicity of equilibria such as the one we are considering.

3. There cannot exist other equilibria in pure strategies than those characterized at points 1 and 2.

We must now consider what happens if either a) more than n value-maximizing members vote "no" and the others vote "yes", or b) more than k value-maximizing members vote "yes" and the rest vote "no".

a) If n - h value-maximizing members choose $m_z = 0$, and k + h value-maximizing members choose $m_j = 1$, $n \ge h > 0$, the best response of M_i , $i \ne j, z$, is to choose $m_i = 0$ because M_i is never pivotal when v = 1 while he may be pivotal when v = -1. This happens in the case where h of those n+h value-maximizing members who choose $m_j = 1$ if uninformed, are in fact informed. As this is true for any h > 0, we are back to the case examined at point 1 above.

b) If n + h value-maximizing members choose $m_z = 0$, and k - h value-maximizing members choose $m_j = 1$, $k \ge h > 1$, the best response of M_i , $i \ne j, z$, is to choose $m_i = 1$ because M_i is never pivotal when v = -1 while he may be pivotal when v = 1. This happens in the case where h of those n + h value-maximizing members who choose $m_z = 0$ if uninformed, are in fact informed. As this is true for any h > 1, we are back to the case examined at point 2 above.

ii₂) In the case of n-k biased members (n > k > 0) and n+1+k value-maximizing members, there may exist equilibria where some or all of the M members choose the same mixed strategy.

In an equilibrium where all the M members choose the same mixed strategy, the value of such strategy, \overline{m} , can be derived by maximizing the function V(m) defined as follows for $m \in [0, 1]$

$$V(m) = E(v) = \frac{1}{2}[Y(\cdot|v=1) - Y(\cdot|v=-1)] \text{ when}$$

$$m_i = m_j = \overline{m}, \forall j \neq i \ i, j = 1, 2..., n + k + 1$$

$$B_z \text{ always votes "yes" } z = 1, .., n - k$$

Given that (a) all M players have identical payoffs equal to E(v), (b) the game is symmetric with respect to the M players, and (c) the number of the M members exceeds that of the Bmembers by more than one, the same argument as in Proposition 1, point iii can be used to prove that the maximization of V(m) has an interior solution $\overline{m} < 1/2$, and that \overline{m} represents the best reply of the M members. Considering that any B member is following his dominant strategy this then represents an equilibrium. We have solved this problem for the case of n = 2, with one member of type B (hence, B_1 always votes "yes") and the remaining four members of type M. The solution of the problem, evaluated at $\alpha = \frac{1}{2}$, is:

$$m_1 = m_2 = m_3 = m_4 = \frac{5 - \sqrt{13}}{6}$$

yielding E(v) = .384973.

Analogously to the third and fourth case of proposition 1, there may also exist equilibria in which some of the M members choose $m_j = 0$, and the others choose the same mixed strategy, or even equilibria where part of the M members choose $m_z = 1$. We do not characterize such equilibria but we show at point iii) that, whenever they exist, they are suboptimal.

iii) The equilibria in pure strategies are optimal, other equilibria are suboptimal Recall that

Recall that

$$E(v) = \frac{1}{2} [Y(\cdot|v=1) - Y(\cdot|v=-1)]$$

= $\frac{1}{2} [1 - Y(\cdot|v=-1) - (1 - Y(\cdot|v=1))].$

where $Y(\cdot|v=-1) + (1 - Y(\cdot|v=1))$ is the probability of making the wrong decision.

In the unique equilibrium of the case with n biased members and n+1 value-maximizing members (point i), as well as in the pure strategy equilibrium of the case with n-k biased members (n > k > 0) and n+1+k value-maximizing members (point ii₁), $Y(\cdot|v=-1) = 0$ and $Y(\cdot|v=1)$ is equal to the probability that at least one of the n+1 members who choose $m_i = 0$ is informed. Then in both cases E(v) is equal to

$$\frac{1}{2}\sum_{i=0}^{n}\frac{(n+1)!}{i!(n+1-i)!}\alpha^{n+1-i}(1-\alpha)^{i} = \frac{1}{2}\left[1-(1-\alpha)^{n+1}\right] = E(v)^{*}$$
(2)

implying that these equilibria are optimal.

In order to prove that the equilibria where unbiased members choose mixed strategies are suboptimal, an analogous argument to the one used in point v) of Proposition 1 can be applied. Consider the equilibrium introduced as an example at point ii)₂ above. Each the M members, individually taken, is in fact indifferent among any $m_i \in [0, 1]$. Let M_1 switch to $m_1 = 0$. E(v) is not modified but this is no longer an equilibrium because $m_j = \frac{5-\sqrt{13}}{6}$ was calculated so as to make unbiased members playing mixed strategies indifferent among $m_j \in [0, 1]$ when there were four of them, while now there are only three M members playing $m_j = \frac{5-\sqrt{13}}{6}$. This implies that one of the members who are still choosing $m_j < \frac{1}{2}$, say M_2 , has as his best response $m_2 = 1$ because he is now pivotal with a higher probability in v = 1 than in v = -1. This in turn implies that, by switching to $m_2 = 1$, he can raise E(v) above

.384973. The resulting situation will not be an equilibrium (as m_j is unaltered for those who still play the mixed strategy, the probability that one of them is pivotal is higher in v = -1 than in v = 1) meaning that there is another member, say M_3 who can further raise E(v) by switching to $m_3 = 0$. Since now there is a B_1 always voting "yes", M_1 and M_3 choosing $m_1 = m_3 = 0$ and M_2 choosing $m_2 = 1$, we know from point ii)₁1 that the best response of M_4 is to choose $m_4 = 0$. But, by doing so, M_4 further raises E(v) while reaching an equilibrium of type ii)₂.

Clearly the same procedure applies to any equilibrium where the M members adopt the same mixed strategy, independently of the size of the board, and of the proportion to the B members. It also applies to possible equilibria in which some of the M members choose $m_j = 0$, and the others choose the same mixed strategy, or to equilibria where part of the Mmembers choose $m_z = 1$. First of all observe that for such an equilibrium to be established it must be at least n = 2 and that the agents adopting a mixed strategy must choose the same m_k . Moreover there must be at least three agents choosing the mixed strategy. It is immediate to verify that there cannot be only two of them, because if there is at least one M member choosing a pure strategy and only one M member choosing a mixed strategy, the best response of the remaining member is a pure strategy. Whenever there are more than one agent using the same mixed strategy, however, we can consider the situation where one of these agents switches from his mixed equilibrium strategy to a pure one. This would not represent an equilibrium and E(v) could be improved by having another agent moving to his best response. By applying this method recursively the equilibria in pure strategies are reached.

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