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Abstract

We investigate how market power shapes industrialization and underdevelopment. We propose a general equilibrium model that allows for different market structures and in which firms choose between a constant-returns-to-scale technology and an increasing-returns-to-scale technology, that is more productive at larger scales. We depart from the traditional Big Push paradigm by showing that market power alone, without relying on coordination failures, is sufficient to generate underdevelopment. Different equilibria arise as the degree of competition in the economy varies, and more competitive economies are characterized by industrialization, *i.e.*, the adoption of the increasing-returns-to-scale technology. These findings suggest a reinterpretation of Big Push theory: public intervention should also foster competition, and not only investment coordination per se.

Keywords: Big Push, Market power, Industrialization, Underdevelopment, Increasing returns to scale.

JEL Code: O14, D43, D51

1 Introduction

The Big Push theory of industrialization was first formulated by Rosenstein-Rodan (1943), who illustrated it through a celebrated example of a shoe factory. Consider an underdeveloped economy: if one firm, the shoe factory, industrializes (i.e., adopts a new technology characterized by higher productivity at a large scale of production), it will make losses due to insufficient demand. Therefore, the firm will decide not to adopt the new technology. In contrast, if many firms industrialize simultaneously, they will make positive profits because the additional income generated by full industrialization increases the size of the market in all sectors. This framework justifies a Big Push policy in which a central authority coordinates simultaneous investment across many sectors of the economy to achieve full industrialization.

This paper proposes a different mechanism through which a lack of industrialization and underdevelopment can arise: market structure. The key insight is straightforward. Firms with market power may find it more profitable to set higher prices, thereby

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reducing aggregate demand and making the adoption of the less productive technology optimal. It is important to remark that this mechanism does not rely on coordination failures, but is driven solely by the degree of competition in the economy. By delivering a reinterpretation of Big Push theory, this paper argues that public intervention should also target the enhancement of competition and not only investment coordination per se. Our analysis departs from the static model of Big Push proposed by Murphy, Shleifer, and Vishny (1989), which is elegant in its simplicity while capturing the key ingredients of the original Big Push theory as expressed by Rosenstein-Rodan (1943).

A growing empirical literature has documented that imperfect competition is a pervasive feature of both advanced and developing economies. Using markups as a measure of market power, the evidence shows a particularly strong rise in advanced economies (De Loecker, Eeckhout, and Unger, 2020; Autor et al., 2020), while their role in developing countries has also received increasing attention. De Loecker and Eeckhout (2018) highlight that developing economies display differences relative to developed countries. However, while countries in South America show a higher level of markups compared to ones in Europe, North America, and Oceania, other developing countries in Asia and Africa do not display such a clear pattern. Similar findings are also confirmed by Heresi (2024) and Díez, Leigh, and Tambunlertchai (2018). Even if the evidence on whether market power is systematically higher in developing countries remains mixed, overall these contributions suggest that imperfect competition is a relevant feature of some developing economies and that understanding its consequences for industrialization is an important research question. This paper identifies a clear channel through which market power can affect industrialization and, consequently, economic development.[1] Another interesting finding is documented by Díez, Fana, and Villegas-Sánchez (2021), who identify a new stylized fact: "markups first decrease with firm size and only when a (fairly large) size threshold is reached, markups start increasing with firm size". Heresi (2024) also finds that "larger firms charge lower markups, except at the very top of the size distribution", conducting a separate analysis for developing economies, emerging markets, and Latin America. The negative relationship between size and markups, which emerges below the size threshold identified by Díez, Fana, and Villegas-Sánchez (2021), may be consistent with the mechanism proposed in this paper, even if our analysis does not deal explicitly with markups, as we further discuss in the conclusions. Indeed, small firms active in less competitive markets find it more profitable to set higher prices and adopt the less productive technology, thereby impeding industrialization and development.

We consider an economy with n consumption goods and a representative agent with a CES utility function. The only input in the economy is labor and each good can be produced by a *cottage method*, characterized by constant returns to scale, or by an *industrial method*, characterized by increasing returns to scale (due to the presence of a fixed cost). Furthermore, to capture different market structures and varying degrees of competition across sectors, we allow for markets having either a competitive fringe of small firms with access only to the cottage method alongside a monopolist with access to both technologies, or only a monopolist. In contrast, Murphy, Shleifer, and Vishny (1989) assume that in each market there is a competitive fringe and that the small firms adopting the cottage method behave competitively, while the monopolist engages in limit pricing. Despite the fact that they consider an imperfectly competitive setting, to obtain equilibria characterized by underdevelop-

[1] Even if this is not relevant to our static analysis, both Díez, Leigh, and Tambunlertchai (2018) and De Loecker and Eeckhout (2018) show that the recent upward trend in global market power is led by developed economies.

ment, they need to introduce a wage premium for the labor employed in firms using the industrial method. Due to the wage premium, firms cannot break even when only a few of them adopt the industrial method because they face insufficient demand. Despite its usefulness, the wage premium can be seen as an *ad hoc* assumption because, from a general equilibrium perspective, it can be justified only by defining consumers' preferences over the method of production and not only over the consumption set. Relative to Murphy, Shleifer, and Vishny (1989), our contribution is twofold. First, we dispense with the wage premium assumption. Second, we show that market power alone —through its effect on prices— is sufficient to generate underdevelopment as an equilibrium outcome, without relying on coordination failures.

Our analysis is articulated in three parts. In the first part, we define a production economy with increasing returns to scale. To deal with the non-convexities arising from the industrial method, we allow firms to follow general pricing rules rather than the classical profit maximization behavior.[2] Specifically, we consider loss-free equilibria in which consumers maximize utility subject to their budget constraint, all markets clear, and prices are such that all firms make non-negative profits. In this setting, we explore the existence of loss-free equilibria characterized by *full industrialization*, at which all firms adopt the industrial method, and loss-free equilibria characterized by *partial industrialization*, at which only a few firms industrialize —the case we interpret as underdevelopment. We show in Proposition 1 that when some relative prices are sufficiently low, there are loss-free equilibria characterized by partial industrialization, i.e., some firms would make negative profits if they adopted the industrial method. By contrast, when all relative prices are close or equal to one, loss-free equilibria are characterized by full industrialization. Proposition 1 clearly shows that prices play an important role in determining industrialization or underdevelopment. This feature is hidden in the analysis of Murphy, Shleifer, and Vishny (1989) because, as a consequence of their assumption on the market structure, all consumption goods have the same price.[3]

In the second part, we deal with the main weakness of using loss-free equilibria. Since firms "accept" any price which guarantees non-negative profits, there are too many equilibrium outcomes giving rise to a significant indeterminacy. To overcome this issue, we introduce a game à la Bertrand with the aim of using its Nash equilibria as a rationale to restrict our analysis to a small set of game-theoretically founded loss-free equilibria. That is, the prices that arise at Nash equilibria are endogenously determined by the firms' strategic choices. We assume that in each of the first k sectors, a monopolist competes against a competitive fringe, as in Murphy, Shleifer, and Vishny (1989), while in each of the remaining $n-k$ sectors only a monopolist operates. This latter generalization requires a CES utility function, since Cobb-Douglas demand functions do not admit a well-defined solution to the monopolist's profit maximization problem. The first result we obtain in this part, Proposition 2, shows that each allocation and price supported by Nash equilibrium strategies is a loss-free equilibrium. This confirms that Nash equilibria can be used to restrict attention to a subset of loss-free equilibria. Since by varying k we can consider different market structures and change the overall degree of competition in the economy, the following propositions characterize the Nash equilibria for different k. Proposition 3 shows that when all markets have

[2] Pricing rules in general equilibrium models with increasing returns to scale are discussed extensively in the literature. See, for instance, Cornet (1988), Vohra (1988), and Bonnisseau (1992), and Villar (2000) for a textbook treatment.

[3] Clearly, this was done to obtain a more tractable model as pointed out explicitly by Ciccone (2008).

a competitive fringe, $k = n$, a Nash equilibrium featuring full industrialization exists. We observe that this corresponds to the highest degree of competition in the economy since there is a competitive fringe in all markets. Proposition 4 shows that when $k < n$ —we actually restrict to the case $k = n-1$—, consumption goods have different prices and it is possible to obtain Nash equilibria characterized by partial industrialization corresponding to the case of underdevelopment. We observe that the results of these propositions are obtained only for some values of the elasticity of substitution. This is needed both for tractability, in order to work with closed-form solutions, and because for some values of σ the monopolist's profit maximization problem does not admit a solution when it does not face a competitive fringe. Nonetheless, despite some of our results lacking full generality, they clearly establish that underdevelopment can arise as a Nash equilibrium outcome, showing that the degree of market competition, and not only coordination failures, can be a key determinant of industrialization or underdevelopment.

In the last part of the paper, we exploit the fact that we work within a general equilibrium model to make some welfare considerations based on Pareto optimality. We first provide an example of a loss-free equilibrium characterized by partial industrialization and show that it is Pareto optimal. The reason for this result is intuitive. When the endowment of labor in the economy is just sufficient to industrialize all sectors, the higher productivity obtained by using the industrial method does not lead to significantly higher production in all sectors of the economy. Therefore, it is more efficient to industrialize just one sector which can use a higher amount of labor and fully exploit the higher productivity achievable at large scale. This insight is formalized in Proposition 6 where it is shown that if the endowment of labor is sufficiently high, then the loss-free equilibrium with full industrialization is always Pareto optimal.[4] This welfare result supports government intervention aimed at achieving full industrialization, but only when it represents a Pareto improvement over the actual equilibrium of the economy.

We conclude by emphasizing that our approach differs fundamentally from other modern formalizations of Big Push theory. Murphy, Shleifer, and Vishny (1989) and Matsuyama (1991), for instance, consider models with a multiplicity of equilibria that are Pareto rankable. This setting provides a foundation for Big Push theory based on coordination failures, which can be addressed through a coordinated investment policy. Our paper takes a different route. In the Bertrand game, we also obtain different Nash equilibria that are Pareto rankable, but each equilibrium arises under a different market structure. That is, underdevelopment does not stem from coordination failures, but is a direct consequence of a market structure lacking competition. Our setting thus provides a justification for government intervention, but its aim is not to solve a coordination failure —rather, to change the market structure by increasing competition. One could think, for instance, of public investment across sectors aimed at removing barriers to entry.[5] We also note that a role for market power in the Big Push theory was already acknowledged by some of the earlier contributions such as Fleming (1955) and Scitovsky (1954). More recently, De Fontenay (2004) studies the role of market power in the Big Push theory in a model with intermediate products.

The paper is organized as follows. In Section 2, we introduce the model. In Section 3, we show the existence of loss-free equilibria characterized by partial industrializa-

[4]It is worth remarking that results on Pareto optimality extend to the analysis of Murphy, Shleifer, and Vishny (1989) in the model without wage premium but it cannot be applied to the model with wage premium because in such a model the representative consumer has a different utility function.

[5]Parente and Prescott (1994) study the relationship between barriers to entry and development.

tion, which describes underdevelopment. In Section 4, we define the Bertrand game and study the features of Nash equilibria under different market structures. In Section 5, we study the Pareto optimality of loss-free equilibria. In Section 6, we offer concluding remarks and directions for further research. All proofs can be found in the Appendix.

2 Model

We introduce a production economy with increasing returns to scale closely related to Murphy, Shleifer, and Vishny (1989). The two main departures from their framework are the use of a CES utility function and the absence of a wage premium. Furthermore, unlike Murphy, Shleifer, and Vishny (1989), who base their analysis on Nash equilibrium alone, we begin by adopting the loss-free equilibrium as our equilibrium concept to accommodate the non-convexities arising from the industrial method within a general equilibrium framework.

We consider a production economy with a representative consumer (henceforth simply consumer), n firms indexed by j, $j = 1, \ldots, n$, and $n+1$ commodities indexed by h, $h = 1, \ldots, n+1$. Commodities $h = 1, \ldots, n$ are consumption goods while commodity $n+1$ is labor.

We assume that the consumer's consumption set is given by $\mathbb{R}^n_+$. A consumption bundle is a vector $x = (x_1, \ldots, x_n)$ in $\mathbb{R}^n_+$, whose elements are amounts of consumption good, i.e., x_h is the quantity of consumption good h. We do not include labor in the commodity bundle because we assume that it is inelastically supplied to the market. The price of commodity h is denoted by p_h and the price vector $p = (p_1, \ldots, p_n)$ belongs to $\mathbb{R}^n_{++}$. The price of labor is not included in the price vector because its price is normalized to 1 throughout the paper.

Consumer's preferences are represented by the following constant elasticity of substitution (CES) utility function

$$u(x) = \left(\sum_{h=1}^{n} \frac{1}{n} x_h^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

with $0 < \sigma < \infty$. The consumer is endowed with nL units of labor, which constitutes the economy's only initial endowment. The consumer owns all the firms in the economy. We denote the profit of firm j by π_j. Hence, the income of the consumer, which corresponds to the economy's income, is

$$M = \sum_{j=1}^{n} \pi_j + nL.$$

Since the utility function is of CES type, the demand functions of the consumer are

$$x_h(p) = \frac{M}{p_h^\sigma \sum_{i=1}^{n} p_i^{1-\sigma}}, \tag{1}$$

for each $h = 1, \ldots, n$. We assume that each consumption good is produced by just one firm by using labor. Furthermore, without loss of generality, we assume that firm 1 produces consumption good 1, firm 2 produces consumption good 2, and so on.

Each firm j is assumed to possess two technologies to produce the consumption good $h = j$. The first technology, called *cottage method*, displays constant returns to scale and it is represented by the following function

$$f_j^C(z_j) = z_j,$$

with $z_j \geq 0$ the amount of labor employed as input, for each $j = 1, \dots, n$. The second technology, called *industrial method*, displays increasing returns to scale (non-convexities) and it is represented by the following function

$$f_j^I(z_j) = \begin{cases} 0 & \text{if } z_j < F \\ \alpha(z_j - F) & \text{if } z_j \geq F \end{cases},$$

for each $j = 1, \dots, n$, with $\alpha > 1$ the productivity of the industrial method, $z_j \geq 0$ the amount of labor employed as input, and $F > 0$ the fixed costs measured in units of labor. Specifically, F is a fixed cost required to adopt the more productive technology. By combining the two technologies above, we can derive the following production function for firm j

$$f_j(z_j) = \begin{cases} z_j & \text{if } z_j \leq \frac{\alpha}{\alpha-1}F \\ \alpha(z_j - F) & \text{if } z_j > \frac{\alpha}{\alpha-1}F \end{cases}, \tag{2}$$

for each $j = 1, \dots, n$, which associates to each level of labor z_j the maximum amount of output.[6] Figure 1 shows the production function of firm j with labor z_j measured on the horizontal axis and the output $f_j(z_j)$ measured on the vertical axis. As the figure

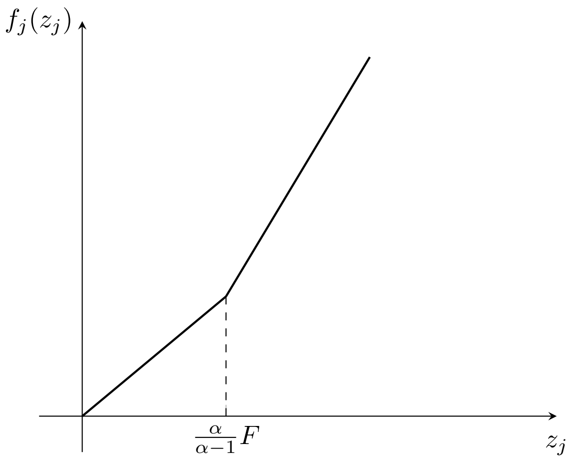

Figure 1: Production set for a generic firm j.

makes clear, the production set displays non-convexities, which are generated by the industrial method. Therefore, we cannot consider the competitive equilibrium because it may fail to exist. To deal with this difficulty, we consider an equilibrium concept where we allow firms to follow more general behavioral rules. For our analysis, it is convenient to assume that firms follow a loss-free behavior which means that given a price vector p, they may produce any quantity of consumption goods as long as profits are non-negative.

[6] In defining the production function, we are implicitly assuming that managers never adopt the cottage method when the industrial method yields higher output.

We denote by $(x, z_1, \ldots, z_n)$ an allocation which is a specification of a consumption bundle for the consumer and the amount of labor employed by each firm $j = 1, \ldots, n$. Note that an allocation provides a complete description of production activities, since the output of commodity h is given by $f_j(z_j)$, with $j = h$, for each $h = 1, \ldots, n$.

Definition 1. A loss-free equilibrium is a pair $(p^*, (x^*, z_1^*, \ldots, z_n^*))$ consisting of a price vector and an allocation such that

(*a*) $x_h^* = x_h(p^*)$, for each $h = 1, \ldots, n$;

(*b*) $p_j^* f_j(z_j^*) - z_j^* \geq 0$, for each $j = 1, \ldots, n$;

(*c*) $x_h^* \leq f_h(z_h^*)$, for each $h = 1, \ldots, n$, and $\sum_{j=1}^n z_j^* \leq nL$.

Condition (a) implies that the commodity bundle x^* maximizes the consumer's utility in the budget set at prices p^*. Condition (b) requires that all firms make non-negative profits. Condition (c) entails that all markets clear. Note that the only difference between the competitive equilibrium and the loss-free equilibrium is condition (b) since at a competitive equilibrium firms are required to maximize profits, whereas here they are only required to make no losses.

3 Loss-free equilibria and underdevelopment

We now investigate whether loss-free equilibria characterized by underdevelopment —in which only a few firms industrialize— can arise in equilibrium. Throughout the paper, we assume that the endowment of labor is sufficient to industrialize all firms in the economy, that is $L > \frac{\alpha}{\alpha-1}F$, otherwise the analysis becomes trivial. Furthermore, we observe that, since the only restriction on firms' behavior is that profits must be non-negative, there are too many loss-free equilibria. For this reason, we focus on two types of equilibria: *full industrialization*, at which all firms adopt the industrial method, and *partial industrialization*, at which the first ℓ firms adopt the industrial method and the last $n - \ell$ firms adopt the cottage method. To further simplify the analysis, we suppose that the first ℓ commodities have a price equal to q and the last $n - \ell$ have a price equal to r.[7] Note that it can be that $q = r$.

The next proposition characterizes these types of loss-free equilibria by introducing a parameter b which can be interpreted as a labor transfer from the last $n - \ell$ firms to the first ℓ firms. That is, if $b = 0$ all firms employ the same amount of labor, L, and if $b > 0$ the first ℓ firms employ a higher amount of labor than the last $n - \ell$ ones.

Proposition 1. Assume that $L > \frac{\alpha}{\alpha-1}F$ and let $\ell = 1, \ldots, n-1$.

(i) For each $b \in [0, (n-\ell)(L - \frac{\alpha}{\alpha-1}F))$, there exists a loss-free equilibrium at which all n firms industrialize and such that

$$(p_1^*, \ldots, p_\ell^*, p_{\ell+1}^*, \ldots, p_n^*) = (q, \ldots, q, r, \ldots, r),$$

$$x^* = (x_1^*, \ldots, x_n^*) = (\alpha(z_1^* - F), \ldots, \alpha(z_n^* - F)),$$

$$(z_1^*, \ldots, z_\ell^*, z_{\ell+1}^*, \ldots, z_n^*) = \left(L + \frac{b}{\ell}, \ldots, L + \frac{b}{\ell}, L - \frac{b}{n-\ell}, \ldots, L - \frac{b}{n-\ell}\right),$$

where $q \geq \frac{L+b/\ell}{\alpha(L+b/\ell-F)}$, $r \geq \frac{L-b/(n-\ell)}{\alpha(L-b/(n-\ell)-F)}$, and $\frac{q}{r} = (\frac{L-b/(n-\ell)-F}{L+b/\ell-F})^{\frac{1}{\sigma}}$.

[7] Proposition 5 on Pareto optimality of loss-free allocations provides a justification to consider only cases where each commodity has prices equal either to q or to r. Note that we can always relabel commodities in order to obtain a price vector where the price of the first ℓ commodities is q and the price of the last $n - \ell$ ones is r.

(ii) For each $b \in [(n-\ell)(L-\frac{\alpha}{\alpha-1}F),(n-\ell)L)$, there exists a loss-free equilibrium at which the first ℓ firms industrialize and the last $n-\ell$ firms adopt the cottage method and such that

$$(p_1^{**},\ldots,p_\ell^{**},p_{\ell+1}^{**},\ldots,p_n^{**})=(q,\ldots,q,r,\ldots,r),$$

$$x^{**}=(x_1^{**},\ldots,x_\ell^{**},x_{\ell+1}^{**},\ldots,x_n^{**})=\left(\alpha\left(z_1^{**}-F\right),\ldots,\alpha\left(z_\ell^{**}-F\right),z_{\ell+1}^{**},\ldots,z_n^{**}\right),$$

$$(z_1^{**},\ldots,z_\ell^{**},z_{\ell+1}^{**},\ldots,z_n^{**})=\left(L+\frac{b}{\ell},\ldots,L+\frac{b}{\ell},L-\frac{b}{n-\ell},\ldots,L-\frac{b}{n-\ell}\right),$$

where $q \geq \frac{L+b/\ell}{\alpha(L+b/\ell-F)}$, $r \geq 1$, and $\frac{q}{r}=(\frac{L-b/(n-\ell)}{\alpha(L+b/\ell-F)})^{\frac{1}{\sigma}}$.

The proposition shows that, given $\ell=1,\ldots,n-1$, different loss-free equilibria are obtained by varying $b \in [0,(n-\ell)L)$. All such equilibria can be partitioned into two types depending on the relative price $\frac{q}{r}$. The first type corresponds to full industrialization and arises when $\frac{q}{r}$ is sufficiently close or equal to 1. In this case, the prices of q and r are close to each other or equal so that all firms face a sufficiently high demand that makes the adoption of the industrial method profitable. The second type corresponds to partial industrialization and arises when $\frac{q}{r}$ is sufficiently lower than one. In this case, firms producing the cheaper consumption goods industrialize because each of them faces a sufficiently high demand which makes it profitable to adopt the industrial method. In contrast, firms producing the more expensive consumption goods adopt the cottage method, as each of them faces a demand too low to cover the fixed cost of industrialization.

While Proposition 1 shows that prices have a key role for the existence of loss-free equilibria with partial industrialization —which we regard as underdevelopment equilibria— it also reveals a significant indeterminacy. Indeed, the large multiplicity of relative price systems supporting different loss-free equilibria makes it unclear which outcome should be selected. Here it is important to observe that in Murphy, Shleifer, and Vishny (1989) the multiplicity of equilibria is obtained for a given price vector and thus arises from coordination failure. Here, instead, multiplicity arises as the price vector changes. This is the reason why we prefer the term underdevelopment instead of "underdevelopment trap". However, in the current setting, it is not clear how the price heterogeneity across commodities supporting partial industrialization can be justified given the symmetry of the model, since all commodities enter the CES utility function symmetrically and all firms have access to the same kind of technologies.

In the next section, we introduce a game to resolve the indeterminacy and to obtain equilibria characterized by underdevelopment as a consequence of market structure.

4 Nash equilibria and underdevelopment

We now introduce a Bertrand game with the aim of using its Nash equilibria as a rationale to restrict our analysis to a subset of game-theoretically founded loss-free equilibria. We assume that in the game each consumption good can be produced by one firm having access to both technologies, which will be referred to as a monopolist, analogously to the economy described in Section 2. Furthermore, the first $k \geq 1$ consumption goods can also be produced by a competitive fringe of small firms which have access only to the cottage method. To simplify the analysis, only monopolists act as players in the game while the small firms in the competitive fringe are assumed to sell their output at a price equal to their marginal cost which is 1.

The variable k is very important because it allows us to consider different market structures. Indeed, for $k = n$, the case considered by Murphy, Shleifer, and Vishny (1989), the economy has the highest degree of competition allowed by our setting since in each market there is a competitive fringe. By decreasing k the level of competition decreases because in such cases some markets are dominated by just one monopolist. In other words, we introduce the possibility of not having a competitive fringe in $n-k$ markets to break the symmetry that characterizes the model of Murphy, Shleifer, and Vishny (1989) and to show that underdevelopment may be caused by some market structures that are not competitive enough.

The Bertrand game is defined as follows. For each $j = 1, \ldots, n$, the strategy set of a monopolist j is $S_j = \{s_j = (t_j, v_j) \in \{C, I\} \times \mathbb{R}_+\}$ with t_j the technology chosen by the monopolist, which can be the cottage method (C) or the industrial method (I), and $v_j \in [0, \infty)$ the price set by the monopolist for the commodity j.[8] Let $S = \prod_{j=1}^{n} S_j$ and $S_{-i} = \prod_{j \neq i} S_j$. Let s and s_{-i} be elements of S and S_{-i} respectively. To understand how the monopolists' profit (payoff) functions are defined, it is important to note that in the first k markets we adopt the tie-breaking rule that when the monopolist and the competitive fringe set the same price, all market demand is served by the monopolist. We then have that monopolist j's profit function is given by

$$\pi_j(s) = \begin{cases} 0 & \text{if } v_j > 1 \\ (v_j - 1)\,\frac{M(s)}{v_j^{\sigma} \sum_{i=1}^{n} v_i^{1-\sigma}} & \text{if } t_j = C \text{ and } v_j \leq 1 \\ \left(v_j - \frac{1}{\alpha}\right)\frac{M(s)}{v_j^{\sigma} \sum_{i=1}^{n} v_i^{1-\sigma}} - F & \text{if } t_j = I \text{ and } v_j \leq 1 \end{cases}, \tag{3}$$

for each $j = 1, \ldots, k$, and monopolist j's profit function is given by

$$\pi_j(s) = \begin{cases} (v_j - 1)\,\frac{M(s)}{v_j^{\sigma} \sum_{i=1}^{n} v_i^{1-\sigma}} & \text{if } t_j = C \\ \left(v_j - \frac{1}{\alpha}\right)\frac{M(s)}{v_j^{\sigma} \sum_{i=1}^{n} v_i^{1-\sigma}} - F & \text{if } t_j = I \end{cases}, \tag{4}$$

for $j = k+1, \ldots, n$, where $M(s)$ is the consumer's income, which is also the economy's income, corresponding to the strategy profile $s \in S$. We now comment on the profit function of a monopolist $j = 1, \ldots, k$ which competes with the competitive fringe. In the first case, monopolist j sells nothing and makes zero profits because it sets a price higher than 1. Therefore, all the market demand for the commodity $h = j$ is served by the competitive fringe. In the second and third cases, monopolist j sets a price lower than or equal to 1 and it serves all the market demand. In contrast, a monopolist $j = k+1, \ldots, n$ always serves all the market demand for the commodity $h = j$ as it does not face any competitive fringe. The game à la Bertrand just described is defined by the set $\Gamma_k(n) = \{(S_1, \pi_1(s)), \ldots, (S_n, \pi_n(s))\}$, for $k = 1, \ldots, n$.

Since the game is based on an economy of the type defined in Section 2, we can determine a price vector and an allocation, for each strategy profile $s \in S$.[9] We denote by $p(s) = (p_1(s), \ldots, p_n(s))$ the price vector associated with a strategy profile $s \in S$, with $p_h(s) = \min\{v_h, 1\}$ for each $h = 1, \ldots, k$ and $p_h(s) = v_h$ for each $h = k+1, \ldots, n$. Next, we denote by $M(s) = \sum_{j=1}^{n} \pi_j(s) + nL$ the economy's income associated with a strategy profile $s \in S$. We note that it cannot be negative as it

[8] Throughout the game-theoretic analysis, we usually refer to a firm j as a monopolist j.

[9] The only difference with the economy defined in Section 2 is the presence of the competitive fringe in the first k sectors. Since for any Nash equilibrium there exists an outcome-equivalent Nash equilibrium in which all production is made by the monopolists, this difference becomes irrelevant for the equilibrium analysis.

corresponds to the sum of the monopolists' revenues. We then denote by $x(s) = (x_1(s), \dots, x_n(s))$ the consumption bundle associated with a strategy profile $s \in S$, with $x_h(s) = \frac{M(s)}{p_h(s)^\sigma \sum_{i=1}^n p_i(s)^{1-\sigma}}$, for each $h = 1, \dots, n$. Furthermore, let $H^C(s)$ denote the set of commodities produced by the cottage method and $H^I(s)$ denote the set of commodities produced by the industrial method, for each $s \in S$. We denote by $z_h(s)$ the amount of labor demanded to produce the consumption good h associated with a strategy profile $s \in S$, with $z_h(s) = x_h(s)$ for each consumption good $h \in H^C(s)$ and with $z_h(s) = \frac{x_h(s)}{\alpha} + F$ for each consumption good $h \in H^I(s)$. Finally, we denote by $y_h(s)$ the total production of consumption good h associated with a strategy profile $s \in S$, with $y_h(s) = z_h(s)$ for each consumption good $h \in H^C(s)$ and with $y_h(s) = \alpha(z_h(s) - F)$ for each consumption good $h \in H^I(s)$. We finally observe that $z_h(s)$ and $y_h(s)$ represent either the labor input and output of the monopolist producing commodity h, or the total labor input and output of the competitive fringe when it produces commodity h with the cottage method. The Lemma in the Appendix shows that in the game all markets clear, for each $s \in S$. That is, $x_h(s) = y_h(s)$, for each $h = 1, \dots, n$, and $\sum_{h=1}^n z_h(s) = nL$, for each $s \in S$. If this were not the case, some rationing rule would be necessary.

We now define the Nash equilibrium for the game $\Gamma_k(n)$.

Definition 2. A strategy profile $\hat{s} \in S$ is a Nash equilibrium for $\Gamma_k(n)$ if $\pi_j(\hat{s}) \geq \pi_j(s_j, \hat{s}_{-j})$, for each $s_j \in S_j$, for each $j = 1, \dots, n$.

The next proposition formalizes the relationship between prices and allocations at Nash equilibria and loss-free equilibria.

Proposition 2. Let $\hat{\hat{s}}$ be a Nash equilibrium of $\Gamma_k(n)$. Then, there exists another Nash equilibrium $\hat{s}$ at which $\hat{v}_j \leq 1$, for each $j = 1, \dots, k$, and such that $p(\hat{s}) = p(\hat{\hat{s}})$ and $(x(\hat{s}), z_1(\hat{s}), \dots, z_n(\hat{s})) = (x(\hat{\hat{s}}), z_1(\hat{\hat{s}}), \dots, z_n(\hat{\hat{s}}))$. Furthermore, the pair $(p^*, (x^*, z_1^*, \dots, z_n^*))$, such that $p^* = p(\hat{s})$ and $(x^*, z_1^*, \dots, z_n^*) = (x(\hat{s}), z_1(\hat{s}), \dots, z_n(\hat{s}))$ is a loss-free equilibrium.

The proposition shows that for any Nash equilibrium there exists another Nash equilibrium where the small firms in the competitive fringe produce nothing and the price and the allocation obtained at such Nash equilibrium are a loss-free equilibrium for the economy described in Section 2. We refer to the vector $(x(\hat{s}), z_1(\hat{s}), \dots, z_n(\hat{s}))$ as a Nash allocation. It should now be clear that Proposition 2 allows us to focus our analysis on Nash allocations because they are a subset of the loss-free allocations which are supported by a price vector determined by the monopolists' equilibrium behaviors in the game.

This proposition together with Proposition 1 suggests that in order to obtain Nash equilibria characterized by partial industrialization we need to consider market structures that can give rise to sufficiently high price differentials across goods. This clarifies why we have ruled out the case with $k = 0$ as in such cases all prices turn out to be equal and all monopolists adopt the industrial method even if there is no competitive fringe in any sector. We highlight that this is the case because the endowment of labor is sufficient to industrialize all sectors in the economy and thus partial industrialization can arise only when labor is allocated in a sufficiently asymmetric way across sectors.

In the next propositions we study the Nash equilibrium of games with different market structures to determine which ones support full industrialization or partial industrialization. We start by considering Nash equilibria characterized by full industrialization when there is a competitive fringe in each sector, that is $k = n$.

Proposition 3. Assume that $L > \frac{\alpha}{\alpha-1}F$ and that $0 < \sigma \leq 1$ or $\sigma = 2$. The strategy profile

$$\hat{s} = \big((\hat{t}_1, \hat{v}_1), \ldots, (\hat{t}_n, \hat{v}_n)\big) = ((I,1), \ldots, (I,1))\,. \tag{5}$$

is a Nash equilibrium for $\Gamma_n(n)$.

The proposition establishes that the Nash equilibrium associated with the more competitive market structure under consideration corresponds to full industrialization when the elasticity of substitution is not too high. If $\sigma = 1$, the utility function is a Cobb-Douglas and we recover a setting analogous to that of Murphy, Shleifer, and Vishny (1989). Since the CES utility function gives rise to more complex calculations than the Cobb-Douglas utility function, we restrict attention to $0 < \sigma \leq 1$ and $\sigma = 2$, values of σ for which we can derive analytically the Nash equilibrium. Nonetheless, a restriction on the elasticity of substitution is also required because for high values of σ consumption goods become close substitutes and monopolists may find it profitable to deviate by undercutting the price as the following example, with $\sigma = 4$, clarifies.

Example 1. Consider an economy with two goods, $n = 2$, such that $\sigma = 4$, $L = 4.5$, $\alpha = 3$, and $F = 2$. In the game $\Gamma_2(2)$, where both monopolists face a competitive fringe, the strategy profile

$$((t_1, v_1), (t_2, v_2)) = ((I,1), (I,1))$$

is not a Nash equilibrium because the strategy $(I, 0.9)$ is a profitable deviation for monopolist 1.

The example suggests that the restriction on the elasticity of substitution is not needed to rule out the adoption of the cottage method — an outcome already ensured by the presence of the competitive fringe — but rather to prevent inter-sector price competition from becoming too intense, which would otherwise give monopolists an incentive to undercut the price set by the competitive fringe. The detailed calculations of this and the subsequent examples can be found in the appendix.

We continue by considering Nash equilibria characterized by partial industrialization when some sectors do not have a competitive fringe.

Proposition 4. Assume that $L > \frac{\alpha}{\alpha-1}F$, that $\sigma = 2$, and that $k = n-1$. Consider the following strategy profile

$$\begin{gathered}\hat{s} = \big((\hat{t}_1, \hat{v}_1), \ldots, (\hat{t}_{n-1}, \hat{v}_{n-1}), (\hat{t}_n, \hat{v}_n)\big) = \\ \left((I,1), \ldots, (I,1), \left(C, \frac{n-1+\sqrt{(n-1)^2+(n-1)\alpha}}{n-1}\right)\right).\end{gathered} \tag{6}$$

If

$$\left(\frac{(2L-F)\alpha}{2(1+\sqrt{\alpha+1})} + F\right)(1+\sqrt{\alpha^2+1}) > \alpha^2(L-F) > \alpha(L-F) + 4F, \tag{7}$$

then:

(*i*) the strategy $\hat{s}$ is a Nash equilibrium for $\Gamma_{n-1}(2)$.

(*ii*) there exists an n sufficiently high such that $\hat{s}$ is no longer a Nash equilibrium for $\Gamma_{n-1}(n)$.

The proposition establishes two important results. Part (i) shows that partial industrialization can be an equilibrium outcome of the game. That is, market power can indeed give rise to underdevelopment. The result in (ii) is more subtle and can be interpreted as a comparative statics exercise. It shows that by increasing the number of sectors with a competitive fringe, the monopolist not facing a competitive fringe finds it profitable to deviate by adopting the industrial method, and thus $\hat{s}$ is no longer a Nash equilibrium. The intuition is straightforward. As n increases, the fraction of sectors with a competitive fringe approaches one, making the economy increasingly competitive. This confirms that a higher level of competition is associated with full industrialization. The next example clarifies that there are economies where condition (7) is satisfied.

Example 2. Consider an economy with n goods such that $\sigma = 2$, $L = 4.5$, $\alpha = 3$, and $F = 2$. This economy satisfies the condition in (7).

It is worth noting that the stricter parameter restrictions required for the partial industrialization result —in particular on n and σ— relative to those for full industrialization, are due to the complexity of calculations arising from the asymmetries when some monopolists adopt the cottage method while others adopt the industrial method. Furthermore, the restriction on $\sigma = 2$ also arises from the fact that for some values of σ the monopolist's profit maximization problem does not admit a solution when it does not face a competitive fringe as the following example clarifies.

Example 3. Consider an economy with two goods, $n = 2$, such that $\sigma < 1$. Consider the game $\Gamma_1(2)$, where only monopolist 1 faces a competitive fringe, and suppose monopolist 1 chooses the strategy $s_1 = (I, 1)$. Under the hypothesis that monopolist 2 chooses the cottage method, its profit maximization problem has no solution.

The results of this section are in line with the insight of Matsuyama (1992): "At the cost of algebraic complexity, one could introduce asymmetry, and for some purposes, such an extension would be necessary."

5 Welfare

In the previous section we have proposed a game-theoretical approach to overcome the indeterminacy arising from the use of loss-free equilibria. In this section we return to the economy defined in Section 2 and, by exploiting the fact that we work in a general equilibrium setting, we use the concept of Pareto optimality to make some welfare considerations. We introduce some additional definitions. A *loss-free allocation* is an allocation $(x^*, z_1^*, \ldots, z_n^*)$ for which there exists a price vector p^* such that the pair $(p^*, (x^*, z_1^*, \ldots, z_n^*))$ is a loss-free equilibrium. A *feasible allocation* is an allocation $(x, z_1, \ldots, z_n)$ that satisfies condition (c) in Definition 1. A *Pareto optimal allocation* is a feasible allocation $(x, z_1, \ldots, z_n)$ for which there does not exist another feasible allocation $(x', z_1', \ldots, z_n')$ such that $u(x') > u(x)$.

We start by investigating whether loss-free allocations characterized by partial industrialization are Pareto optimal or not. Pareto optimality is not immediate to check as there are no general results, like the first welfare theorem, for loss-free equilibria. To deal with this issue we start with an example.

Example 4. Consider an economy with two goods, $n = 2$, such that $L = 2.2$, $\alpha = 2$, and $F = 1$. The loss-free allocation

$$((x_1^{**}, x_2^{**}), z_1^{**}, z_2^{**}) = \left(\left(\frac{2^\sigma 6.8}{2^\sigma + 2}, \frac{6.8}{2^\sigma + 2} \right), \frac{2^\sigma 4.4 + 2}{2^\sigma + 2}, \frac{6.8}{2^\sigma + 2} \right), \tag{8}$$

where firm 1 industrializes and firm 2 adopts the cottage method, is Pareto optimal, for $\sigma \geq 0.98$.

This example is important because it shows that even if the endowment of labor is sufficient to industrialize all firms in the economy, it is Pareto superior to industrialize only one of them. Therefore, loss-free equilibria with partial industrialization can be Pareto optimal in some instances. It is worth noting that this result holds not only when commodities are substitutes ($\sigma > 1$) or independent ($\sigma = 1$) but also when the commodities are complement for a σ sufficiently close to 1 ($0.98 < \sigma < 1$).

The economic intuition behind this result is straightforward. When the endowment of labor in the economy is just sufficient to industrialize both firms, the higher productivity obtained by using the industrial method does not lead to significantly higher production of consumption goods. Therefore, if the elasticity of substitution is not too low, it is preferable, from a Paretian perspective, to industrialize just one firm which can use a higher amount of labor and can better exploit the higher productivity achievable at large scale. In order to confirm this insight we first provide a necessary condition for Pareto optimality.

Proposition 5. If an allocation $(x, z_1, \dots, z_n)$ is Pareto optimal, then $x_i = x_l$ for any pair of consumption goods i and l that are produced with the same technology.

We first note that the proposition implies that at any loss-free equilibrium $(p^*, (x^*, z_1^*, \dots, z_n^*))$, where $(x^*, z_1^*, \dots, z_n^*)$ is Pareto optimal, the price vector p^* must be such that consumption goods produced with the same technology have the same price since otherwise we would have $x_i^*(p^*) \neq x_l^*(p^*)$ for two consumption goods produced with the same technology, which would contradict the proposition. In addition, the proposition allows us to conclude that, for $b \in (0, (n-\ell)(L - \frac{\alpha}{\alpha-1}F))$, any loss-free allocation $(x^*, (z_1^*, \dots, z_n^*))$ with full industrialization characterized in Proposition 1 is not Pareto optimal. Indeed, in all such cases the prices q and r are different but all commodities are produced with the industrial method. In contrast, for $b = 0$, the loss-free allocation with full industrialization can be Pareto optimal as the following proposition shows.

Proposition 6. If the endowment of labor is sufficiently high, then the loss-free allocation

$$(x^*, (z_1^*, \dots, z_n^*)) = ((\alpha(L-F), \dots, \alpha(L-F)), (L, \dots, L)), \tag{9}$$

where all firms industrialize, is Pareto optimal and it Pareto dominates any loss-free allocation $(x^{**}, (z_1^{**}, \dots, z_n^{**}))$ with partial industrialization characterized in Proposition 1.

The intuition behind this result is that, when the endowment of labor is sufficiently high, the industrialization of all firms leads to significantly higher production for all consumption goods than the cottage method. Therefore, in such cases the loss-free allocation with full industrialization in (9) is Pareto optimal and the loss-free allocations with partial industrialization are not Pareto optimal. The proposition also highlights the key role of the endowment of labor as it not only dictates the possibility of an economy to industrialize all firms but also plays a key role in determining the Pareto optimality of full industrialization.

In the theory of Big Push proposed by Rosenstein-Rodan (1943), we may conjecture that the author had in mind an underdeveloped economy where there is abundance of labor and therefore full industrialization is not only feasible but also desirable from a Paretian perspective. Similarly, the equilibrium allocation found by Murphy, Shleifer, and Vishny (1989) in their model without wage premium, which is given by (9), is Pareto optimal only if the initial endowment of labor is sufficiently high.

We conclude with an example showing a Nash allocation characterized by partial industrialization that is not Pareto efficient, so that government intervention is required to move to a Pareto optimal allocation.

Example 5. Consider an economy with two goods, $n = 2$, such that $\sigma = 2$, $L = 4.5$, $\alpha = 3$, and $F = 2$. In the game $\Gamma_1(2)$, where only monopolist 1 faces a competitive fringe, the Nash equilibrium is $\hat{s} = ((I, 1), (C, 3))$. The corresponding Nash allocation,

$$(x(\hat{s}), z_1(\hat{s}), z_2(\hat{s})) = \left(\left(\frac{63}{4}, \frac{7}{4} \right), \frac{29}{4}, \frac{7}{4} \right),$$

is Pareto dominated by the loss-free allocation where both firms industrialize

$$((x_1^*, x_2^*), z_1^*, z_2^*) = ((7.5, 7.5), 4.5, 4.5) .$$

6 Conclusions

This paper proposes a new mechanism through which market power can give rise to underdevelopment. We develop a general equilibrium model in which firms choose between a constant-returns-to-scale technology and a more productive increasing-returns-to-scale technology. Unlike existing models of Big Push theory, our analysis does not rely on coordination failures: different equilibrium outcomes arise under different degrees of market competition. In particular, we show that market power can generate sufficiently high price differentials across goods, leading to partial industrialization, interpreted as underdevelopment. That is, we show that market power alone is sufficient to generate underdevelopment.

To obtain these results, our analysis proceeds in three steps. In the first step, we characterize loss-free equilibria —an equilibrium concept that accommodates the non-convexities arising from the industrial method within a general equilibrium framework. We show that equilibria with partial industrialization exist whenever consumption goods have different prices, and that prices play a key role in determining whether firms industrialize or not. In the second step, we introduce a Bertrand game to discipline the multiplicity of loss-free equilibria. By varying the degree of competition across sectors —captured by the parameter $k \geq 1$— we show that more competitive market structures are associated with full industrialization, while less competitive ones can sustain partial industrialization as a Nash equilibrium outcome. In the third step, we analyze the Pareto optimality of loss-free equilibria. We show that partial industrialization can be Pareto optimal when the labor endowment is relatively low, while full industrialization is always Pareto optimal when the labor endowment is sufficiently high. This latter feature is usually the case in developing countries.

These results deliver a reinterpretation of Big Push theory with important policy implications. The common view, associated with Murphy, Shleifer, and Vishny (1989), argues that underdevelopment is a consequence of coordination failures — hence the use of the term 'underdevelopment trap' that we avoided. According to this view, the Big Push policy should coordinate simultaneous investment across many sectors, for instance, by subsidizing firms that face losses when industrializing. In contrast, our analysis relies on market structure — different equilibria arise under different values of k— and suggests that competition policy can also play a role in promoting industrialization, for instance, by facilitating the entry of new firms or removing barriers to entry in sectors characterized by less competition. We also observe that despite this difference, the reason why industrialization does not take place in our model is essentially the same as that described by Rosenstein-Rodan (1943): the lack of demand. The key difference lies in the source of low demand: in our model it stems from high prices set by firms with market power, while in Rosenstein-Rodan (1943) it is the lack of income in the economy. The channel highlighted in this paper may be especially relevant for developing economies characterized by concentrated product markets, high barriers to entry, and weak competition policies.

The limitations of our analysis suggest directions for further research. Our results on Nash equilibria are obtained only for specific values of the elasticity of substitution, due to the complexity of the calculations involved when consumption goods have different prices under a CES utility function, and because for some values of σ the monopolist's profit maximization problem may not admit a solution when it does not face a competitive fringe, which raises a more general issue of existence for Nash equilibria. Therefore, even if extending the analysis to a broader range of parameter values would strengthen the generality of our conclusions, it seems a challenging task. A more promising avenue for future research may be to consider games describing a different kind of competition among firms. Our game à la Bertrand has the advantage of showing that the link between market power and industrialization (or underdevelopment) arises even in the classical model of price competition. However, the loss-free equilibrium framework considered in this paper is quite general and could in principle accommodate different forms of strategic interaction or market structure. The recent contribution of Wu (2025), for instance, introduces competition in markups among firms in production networks within a general equilibrium setting. Exploring whether such an approach can be adapted to a setting with increasing returns to scale and without intermediate products would allow for an explicit treatment of markups and a more direct connection between the theoretical mechanism proposed in this paper and the empirical evidence on markup dispersion documented by De Loecker and Eeckhout (2018), Díez, Leigh, and Tambunlertchai (2018), and Heresi (2024). We leave this as an open question for future research.

Appendix: Proofs

Proof of Proposition 1. We start by considering the case $b \in [0, (n-\ell)(L - \frac{\alpha}{\alpha-1}F))$. Note that it always holds that $L + \frac{b}{\ell} > \frac{\alpha}{\alpha-1}F$ and that $L - \frac{b}{n-\ell} > \frac{\alpha}{\alpha-1}F$ for $b \in [0, (n-\ell)(L - \frac{\alpha}{\alpha-1}F))$. Therefore, all firms adopt the industrial method. It is immediate to verify that $x_h^* = f_h(z_h^*)$, for each $h = 1, \ldots, n$, and $\sum_{j=1}^{\ell} z_j^* + \sum_{j=\ell+1}^{n} z_j^* = \ell\left(L + \frac{b}{\ell}\right) + (n-\ell)\left(L - \frac{b}{n-\ell}\right) = nL$. That is, condition (c) in Definition 1 is satisfied. Next, $q \geq \frac{L+b/\ell}{\alpha(L+b/\ell-F)}$ implies that

$$\pi_j = p_j^* f_j(z_j^*) - z_j^* = q\alpha\left(L + \frac{b}{\ell} - F\right) - L - \frac{b}{\ell} \geq 0,$$

for each firm $j = 1, \ldots, \ell$. Similarly, $r \geq \frac{L-b/(n-\ell)}{\alpha(L-b/(n-\ell)-F)}$ implies that $\pi_j = p_j^* f_j(z_j^*) - z_j^* \geq 0$, for each firm $j = \ell+1, \ldots, n$. That is, condition (b) in Definition 1 is satisfied.

The economy's income is given by

$$M=\sum_{j=1}^{\ell}\left(\left(q-\frac{1}{\alpha}\right)x_j(p^*)-F\right)+\sum_{j=\ell+1}^{n}\left(\left(r-\frac{1}{\alpha}\right)x_j(p^*)-F\right)+nL.$$

Since $x_h(p^*)=\frac{M}{q^\sigma(\ell q^{1-\sigma}+(n-\ell)r^{1-\sigma})}$, for each $h=1,\dots,\ell$ and $x_h(p^*)=\frac{M}{r^\sigma(\ell q^{1-\sigma}+(n-\ell)r^{1-\sigma})}$, for each $h=\ell+1,\dots,n$, by solving the equation above with respect to M, we obtain the income of the economy under full industrialization

$$M=\frac{\alpha(L-F)nq^\sigma r^\sigma(\ell q^{1-\sigma}+(n-\ell)r^{1-\sigma})}{(n-\ell)q^\sigma+\ell r^\sigma}. \tag{10}$$

Since $\frac{q}{r}=(\frac{L-b/(n-\ell)-F}{L+b/\ell-F})^{\frac{1}{\sigma}}$, it follows that

$$x_h(p^*)=\frac{\alpha n(L-F)r^\sigma}{(n-\ell)q^\sigma+\ell r^\sigma}=\frac{\alpha n(L-F)}{(n-\ell)\left(\frac{L-b/(n-\ell)-F}{L+b/\ell-F}\right)+\ell}=\alpha\left(L+\frac{b}{\ell}-F\right)=x_h^*,$$

for each $h=1,\dots,\ell$ and that

$$x_h(p^*)=\frac{\alpha n(L-F)q^\sigma}{(n-\ell)q^\sigma+\ell r^\sigma}=\frac{\alpha n(L-F)}{(n-\ell)+\ell\left(\frac{L+b/\ell-F}{L-b/(n-\ell)-F}\right)}=\alpha\left(L-\frac{b}{n-\ell}-F\right)=x_h^*,$$

for each $h=\ell+1,\dots,n$. Then, the commodity bundle in x^* corresponds to the competitive demands at price p^*. That is, condition (a) in Definition 1 is satisfied. Since the three conditions in Definition 1 are satisfied, we conclude that the pair $(p^*,(x^*,z_1^*,\dots,z_\ell^*,z_{\ell+1}^*,\dots,z_n^*))$ is a loss-free equilibrium.

For any $b\in[(n-\ell)(L-\frac{\alpha}{\alpha-1}F),(n-\ell)L)$, by following *mutatis mutandis* the same steps above, it is possible to prove that also the pair $(p^{**},(x^{**},z_1^{**},\dots,z_\ell^{**},z_{\ell+1}^{**},\dots,z_n^{**}))$ is a loss-free equilibrium. Note that in such a case, characterized by partial industrialization, the economy's income is given by the solution of the following equation,

$$M=\sum_{j=1}^{\ell}\left(\left(q-\frac{1}{\alpha}\right)x_j(p^{**})-F\right)+\sum_{j=\ell+1}^{n}((r-1)\,x_j(p^{**}))+nL.$$

That is, we obtain the income of the economy under partial industrialization

$$M=\frac{\alpha(nL-\ell F)q^\sigma r^\sigma(\ell q^{1-\sigma}+(n-\ell)r^{1-\sigma})}{\alpha(n-\ell)q^\sigma+\ell r^\sigma}. \tag{11}$$

□

Lemma. For each $s\in S$, we have that $x_h(s)=y_h(s)$, for each $h=1,\dots,n$, and $\sum_{h=1}^n z_h(s)=nL$.

Proof. Consider a strategy profile $s\in S$. From the definition of $x_h(s)$, $z_h(s)$, and $y_h(s)$, it is immediate to observe that

$$x_h(s)=z_h(s)=y_h(s),$$

for each commodity $h\in H^C(s)$, and that

$$x_h(s)=\alpha(z_h(s)-F)=y_h(s),$$

for each commodity $h \in H^I(s)$. Then, the market clearing condition for commodity h is satisfied, for each $h = 1, \ldots, n$. To show that the market clearing condition for labor is also satisfied, we have to keep in mind the following point. Since each consumption good produced with the industrial method is produced by a monopolist, we can interpret $H^I(s)$ as the set of monopolists adopting the industrial method. In contrast, for a commodity $h \in H^C$ with $h = 1, \ldots, k$, $\pi_h(s)$represents the profits of the monopolist if $v_h \leq 1$, and the profits of the competitive fringe if $v_h > 1$; for a commodity $h \in H^C$ with $h = k+1, \ldots, n$, $\pi_h(s)$ always represents the profits of the monopolist, since no competitive fringe is present. From this observation, it follows that

$$nL = M(s) - \sum_{i=1}^{n} \pi_i(s) = M(s) - \sum_{h \in H^I(s)} \pi_h(s) - \sum_{h \in H^C(s)} \pi_h(s) =$$

$$M(s) - \sum_{h \in H^I(s)} p_h(s) x_h(s) + \sum_{h \in H^I(s)} z_h(s) - \sum_{h \in H^C(s)} p_h(s) x_h(s) + \sum_{h \in H^C(s)} z_h(s) =$$

$$M(s) - \sum_{h \in H^I(s)} \frac{p_h(s)^{1-\sigma} M(s)}{\sum_{i=1}^{n} p_i(s)^{1-\sigma}} + \sum_{h \in H^I(s)} z_h(s) - \sum_{h \in H^C(s)} \frac{p_h(s)^{1-\sigma} M(s)}{\sum_{i=1}^{n} p_i(s)^{1-\sigma}} + \sum_{h \in H^C(s)} z_h(s) =$$

$$M(s) - \sum_{h=1}^{n} p_h(s)^{1-\sigma} \frac{M(s)}{\sum_{i=1}^{n} p_i(s)^{1-\sigma}} + \sum_{h=1}^{n} z_h(s) = \sum_{h=1}^{n} z_h(s),$$

where the penultimate equality follows by the fact that $H^C(s) \cup H^I(s) = \{1, \ldots, n\}$. Hence, we can conclude that all markets clear. $\square$

Proof of Proposition 2. Let $\hat{\hat{s}}$ be a Nash equilibrium of the game. Suppose that there exists a monopolist j, with $j = 1, \ldots, k$, such that $\hat{\hat{v}}_j > 1$. Then, commodity $h = j$ is produced by the competitive fringe and the monopolist j produces nothing making zero profits. But then, there exists another Nash equilibrium $\hat{s}$ such that all other monopolists have the same strategy and the monopolist j plays $\hat{s}_j = (C, 1)$ at which it continues to make zero profits. Since this holds for any monopolist $j = 1, \ldots, k$ such that $\hat{\hat{v}}_j > 1$, we conclude that there exists another Nash equilibrium $\hat{s}$ at which $\hat{v}_j \leq 1$, for each $j = 1, \ldots, k$, and such that $p(\hat{s}) = p(\hat{\hat{s}})$ and $(x(\hat{s}), z_1(\hat{s}), \ldots, z_n(\hat{s})) = (x(\hat{\hat{s}}), z_1(\hat{\hat{s}}), \ldots, z_n(\hat{\hat{s}}))$. Let $p^* = p(\hat{s})$ and $(x^*, z_1^*, \ldots, z_n^*) = (x(\hat{s}), z_1(\hat{s}), \ldots, z_n(\hat{s}))$. By the definitions of $x_h(s)$ and of the demand in (1), it follows that

$$x_h^* = x_h(\hat{s}) = \frac{M(\hat{s})}{p_h(\hat{s})^\sigma \sum_{i=1}^{n} p_i(\hat{s})^{1-\sigma}} = x_h(p(\hat{s})) = x_h(p^*),$$

for each $h = 1, \ldots, n$. That is, condition (a) in Definition 1 is satisfied. Next, consider the price p^* and suppose that there exists a monopolist j such that its profits are negative, that is $\pi_j(\hat{s}) < 0$. But then, $s'_j = (C, 1)$ is a profitable deviation for monopolist j, as $0 = \pi_j(s'_j, \hat{s}_{-j}) > \pi_j(\hat{s})$. Since $\hat{s}$ is a Nash equilibrium, a contradiction arises. Hence, $\pi_j(\hat{s}) \geq 0$, for each $j = 1, \ldots, n$. By the definition of $z_h(s)$ and since $\hat{v}_h = p_h(\hat{s})$ and $y_h(\hat{s}) = x_h(\hat{s})$, for each $h = 1, \ldots, n$, it follows that $\pi_j(\hat{s})$ can be written as $p_j(\hat{s}) f_j(z_j(\hat{s})) - z_j(\hat{s})$, for each $j = 1, \ldots, n$. Since $\pi_j(\hat{s}) \geq 0$ for each $j = 1, \ldots, n$, we conclude that $p_j^* f_j(z_j^*) - z_j^* \geq 0$, for each $j = 1, \ldots, n$. That is, condition (b) in Definition 1 is satisfied. Finally, by the Lemma, it follows that $x_h(\hat{s}) = y_h(\hat{s})$, for each $h = 1, \ldots, n$, and $\sum_{h=1}^{n} z_h(\hat{s}) = nL$. But then, $x_h^* = f_h(z_h^*)$ for each $h = 1, \ldots, n$ and $\sum_{j=1}^{n} z_j^* = nL$. That is, condition (c) in Definition 1 is satisfied. Hence, $(p^*, (x^*, z_1^*, \ldots, z_n^*))$ is a loss-free equilibrium for the economy described in Section 2. $\square$

Proof of Proposition 3. At $\hat{s}$ there is full industrialization. Setting $q = r = 1$ in equation (10), we obtain

$$M(\hat{s}) = \alpha(L - F)n.$$

Then, monopolist j's profit at the Nash equilibrium is

$$\pi_j(\hat{s}) = \left(\hat{v}_j - \frac{1}{\alpha}\right) \frac{M(\hat{s})}{\hat{v}_j^{\sigma} \sum_{i=1}^{n} \hat{v}_i^{1-\sigma}} - F = \alpha(L - F) - L, \tag{12}$$

which is positive because $L > \frac{\alpha}{\alpha-1}F$, for each $j = 1, \dots, n$. We now show that no monopolist j has a profitable deviation. We proceed by contradiction and suppose that a monopolist j has a profitable deviation $s'_j = (C, v'_j)$ at which the cottage method is used. But then, it is immediate to see that for any $v'_j \geq 0$ the profit is non positive. Hence, s'_j is never a profitable deviation, regardless of σ, a contradiction. Consider next a deviation $s'_j = (I, v'_j)$, with $v'_j \neq 1$, at which the industrial method is adopted. At $(s'_j, \hat{s}_{-j})$ there is full industrialization. Setting $q = 1$, $r = v'_j$, $\ell = n - 1$ in equation (10), we obtain

$$M(s'_j, \hat{s}_{-j}) = \frac{\alpha(L - F)n v_j'^{\sigma} \left(n - 1 + v_j'^{1-\sigma}\right)}{1 + (n - 1)v_j'^{\sigma}}. \tag{13}$$

Then, monopolist j's profit at the strategy profile $(s'_j, \hat{s}_{-j})$ is

$$\pi_j(s'_j, \hat{s}_{-j}) = \left(v'_j - \frac{1}{\alpha}\right) \frac{\alpha(L - F)n}{1 + (n - 1)v_j'^{\sigma}} - F = \left(\alpha v'_j - 1\right) \frac{(L - F)n}{1 + (n - 1)v_j'^{\sigma}} - F. \tag{14}$$

By taking the derivative of $\pi_j(s'_j, \hat{s}_{-j})$ with respect to v_j we obtain

$$\frac{\partial \pi_j(s'_j, \hat{s}_{-j})}{\partial v_j} = \frac{\alpha(L - F)n((n - 1)v_j'^{\sigma} + 1) - (n - 1)\sigma v_j'^{\sigma-1}(\alpha v'_j - 1)(L - F)n}{\left(1 + (n - 1)v_j'^{\sigma}\right)^2},$$

which can be rewritten as

$$\frac{(L - F)n(n - 1)\left(\alpha(v_j'^{\sigma} + \frac{1}{n-1}) - \sigma v_j'^{\sigma-1}(\alpha v'_j - 1)\right)}{\left(1 + (n - 1)v_j'^{\sigma}\right)^2}.$$

Since $L > F$ and $n \geq 2$, it follows that $\frac{\partial \pi_j(s'_j, \hat{s}_{-j})}{\partial v_j} > 0$ if and only if

$$\alpha v_j'^{\sigma}(1 - \sigma) + \frac{\alpha}{n - 1} + \sigma v_j'^{\sigma-1} > 0,$$

which is always satisfied for $\sigma \leq 1$. But then, any $v'_j < 1$ cannot be a profitable deviation as $\frac{\partial \pi_j(s'_j, \hat{s}_{-j})}{\partial v_j} > 0$, for $\sigma \leq 1$. Moreover, monopolist j makes zero profit due to the presence of the competitive fringe for any $v'_j > 1$. Hence, s'_j is not a profitable deviation for any $\sigma \leq 1$, a contradiction. We next consider the case for $\sigma = 2$. If $s'_j = (I, v'_j)$, with $v'_j \neq 1$, is a profitable deviation, then $\pi_j(s'_j, \hat{s}_{-j}) > \pi_j(\hat{s})$, which implies

$$\left(\alpha v'_j - 1\right) \frac{(L - F)n}{1 + (n - 1)v_j'^2} - F > \alpha(L - F) - L$$

by equations (12) and (14). By some simplifications, we can obtain

$$\left(\alpha v'_j - 1\right) \frac{n}{(n - 1)v_j'^2 + 1} + 1 - \alpha > 0$$

which can be written as the following second degree inequality

$$(\alpha-1)(n-1)v_j'^2-\alpha n v_j'+n+\alpha-1<0$$

The inequality is satisfied for $1<v_j'<\frac{n+\alpha-1}{\alpha(n-1)-n+1}$, which means that the deviation s_j' would be profitable in this range. However, given the presence of the competitive fringe, if $v_j'>1$, monopolist j sells nothing and makes zero profits. Hence, s_j' is not a profitable deviation for $\sigma=2$, a contradiction. We then conclude that $\hat{s}$ is a Nash equilibrium for $\sigma\leq 1$ and $\sigma=2$. $\square$

Proof of Example 1. The profit of monopolist 1 at the strategy profile $((I,1),(I,1))$ can be calculated using equation (12). We then have

$$\pi_1((I,1),(I,1))=\alpha(L-F)-L=3.$$

Similarly, the profit of monopolist 1 when its strategy is $(I,0.9)$ can be calculated using equation (14). We then have

$$\pi_1((I,0.9),(I,1))=(\alpha v_1-1)\,\frac{(L-F)n}{(n-1)v_1^{\sigma}+1}-F=(3\cdot 0.9-1)\,\frac{(4.5-2)2}{0.9^4+1}-2=3.13$$

Hence, the strategy profile $((I,1),(I,1))$ is not a Nash equilibrium as monopolist 1 has a profitable deviation $(I,0.9)$. $\square$

Proof of Proposition 4. We start by proving the statement in (i). For $n=2$, the Nash equilibrium $\hat{s}$ is

$$\left((\hat{t}_1,\hat{v}_1),(\hat{t}_2,\hat{v}_2)\right)=\left((I,1),\left(C,1+\sqrt{1+\alpha}\right)\right).$$

At $\hat{s}$ there is partial industrialization. Setting $q=1$ and $r=\hat{v}_2$, we do not substitute the numerical value of $\hat{v}_2$ to avoid cumbersome calculations, $n=2$, and $\ell=1$ in equation (11), we obtain

$$M(\hat{s}_1,\hat{s}_2)=\frac{\alpha(2L-F)\hat{v}_2^2\left(1+\hat{v}_2^{-1}\right)}{\alpha+\hat{v}_2^2},$$

Then, monopolist 1's profit at the Nash equilibrium is

$$\pi_1(\hat{s}_1,\hat{s}_2)=\left(1-\frac{1}{\alpha}\right)\frac{M(\hat{s}_1,\hat{s}_2)}{(1+\hat{v}_2^{-1})}-F=\left(1-\frac{1}{\alpha}\right)\frac{\alpha(2L-F)\hat{v}_2^2}{\alpha+\hat{v}_2^2}-F,$$

and monopolist 2's profit at the Nash equilibrium is

$$\pi_2(\hat{s}_1,\hat{s}_2)=(\hat{v}_2-1)\,\frac{M(\hat{s}_1,\hat{s}_2)}{\hat{v}_2^2(1+\hat{v}_2^{-1})}=(\hat{v}_2-1)\,\frac{\alpha(2L-F)}{\alpha+\hat{v}_2^2}. \tag{15}$$

Note also that the condition $L>\frac{\alpha}{\alpha-1}F$ implies that $\pi_1(\hat{s}_1,\hat{s}_2)>0$ as $\hat{v}_2^2=2+\alpha+2\sqrt{1+\alpha}$.

We now show that neither monopolist 1 nor monopolist 2 has a profitable deviation. We proceed by contradiction and we start by supposing that monopolist 1 has a profitable deviation $s_1'=(C,v_1')$ at which the cottage method is used. But then, it is immediate to see that for any $v_1'\geq 0$ the profit is non positive. Hence, s_1' is not a profitable deviation, a contradiction. Consider next a deviation $s_1'=(I,v_1')$, with $v_1' \neq 1$, at which the industrial method is adopted. At $(s_1', \hat{s}_2)$ there is partial industrialization. Setting $q = v_1'$ and $r = \hat{v}_2$, $n = 2$, and $\ell = 1$ in equation (11), we obtain

$$M(s_1', \hat{s}_2) = \frac{\alpha(2L - F)v_1'^2 \hat{v}_2^2 \left(v_1'^{-1} + \hat{v}_2^{-1}\right)}{\alpha v_1'^2 + \hat{v}_2^2},$$

Then, the monopolist 1's profit at the strategy profile $(s_1', \hat{s}_2)$ is

$$\pi_1(s_1', \hat{s}_2) = \left(v_1' - \frac{1}{\alpha}\right) \frac{M(s_1', \hat{s}_2)}{v_1'^2(v_1'^{-1} + \hat{v}_2^{-1})} - F = \left(v_1' - \frac{1}{\alpha}\right) \frac{\alpha(2L - F)\hat{v}_2^2}{\alpha v_1'^2 + \hat{v}_2^2} - F.$$

If s_1' is a profitable deviation, then $\pi_1(s_1', \hat{s}_2) > \pi_1(\hat{s}_1, \hat{s}_2)$ which implies

$$\left(v_1' - \frac{1}{\alpha}\right) \frac{\alpha(2L - F)\hat{v}_2^2}{\alpha v_1'^2 + \hat{v}_2^2} - F > \left(\hat{v}_1 - \frac{1}{\alpha}\right) \frac{\alpha(2L - F)\hat{v}_2^2}{\alpha \hat{v}_1^2 + \hat{v}_2^2} - F$$

by the results above. By some simplifications and by substituting $\hat{v}_1 = 1$, we can obtain

$$\frac{\alpha v_1' - 1}{\alpha v_1'^2 + \hat{v}_2^2} > \frac{\alpha - 1}{\alpha + \hat{v}_2^2}$$

which can be written as the following second degree inequality

$$(\alpha - 1)v_1'^2 - (\hat{v}_2^2 + \alpha)v_1' + \hat{v}_2^2 + 1 < 0$$

It is immediate to verify that the inequality is satisfied, which means that deviation s_1' is profitable, for $1 < v_1' < \frac{\hat{v}_2^2+1}{\alpha-1}$. However, given the presence of the competitive fringe in the market of commodity 1, if $v_1' > 1$, then monopolist 1 sells nothing and makes zero profits. Hence, s_1' is not a profitable deviation, a contradiction. We then conclude that monopolist 1 has no profitable deviation. We now suppose that monopolist 2 has a profitable deviation $s_2' = (C, v_2')$, with $v_2' \neq 1 + \sqrt{1 + \alpha}$ at which the cottage method is used. At $(\hat{s}_1, s_2')$ there is partial industrialization. Setting $q = 1$, $r = v_2'$, $n = 2$, and $\ell = 1$ in equation (11), we obtain

$$M(\hat{s}_1, s_2') = \frac{\alpha(2L - F)v_2'^2 \left(1 + v_2'^{-1}\right)}{\alpha + v_2'^2},$$

Then, the monopolist 2's profit at the strategy profile $(\hat{s}_1, s_2')$ is

$$\pi_2(\hat{s}_1, s_2') = (v_2' - 1) \frac{M(\hat{s}_1, s_2')}{v_2'^2(1 + v_2'^{-1})} = (v_2' - 1) \frac{\alpha(2L - F)}{\alpha + v_2'^2}$$

and the first order condition associated to the unconstrained profit maximization problem is

$$\frac{\partial \pi(\hat{s}_1, s_2')}{\partial v_2} = \frac{\alpha(2L - F)\left(\alpha + 2v_2' - v_2'^2\right)}{(\alpha + v_2'^2)^2} = 0.$$

This simplifies to a second degree equation

$$v_2'^2 - 2v_2' - \alpha = 0$$

whose positive solution is $\hat{v}_2 = 1 + \sqrt{1 + \alpha}$. Since $\frac{\partial \pi(\hat{s}_1, s_2')}{\partial v_2} > 0$, for each $v_2' \in [0, \hat{v}_2)$, and $\frac{\partial \pi(\hat{s}_1, s_2')}{\partial v_2} < 0$, for each $v_2' \in (\hat{v}_2, \infty)$, we conclude that $\hat{v}_2$ corresponds to the maximum point of the monopolist 2 profit function when it adopts the cottage method and monopolist 1 plays $\hat{s}_1$. Then, s_2' cannot be a profitable deviation, a contradiction. Consider next a deviation $s_2' = (I, v_2')$ at which the industrial method is adopted. At $(\hat{s}_1, s_2')$ there is full industrialization. Setting $q = 1$, $r = v_2'$, $n = 2$, and $\ell = 1$ in equation (10), we obtain

$$M(\hat{s}_1, s_2') = \frac{\alpha 2(L-F)v_2'^2\left(1 + v_2'^{-1}\right)}{1 + v_2'^2}.$$

Then, the monopolist 2's profit at the strategy profile $(\hat{s}_1, s_2')$ is

$$\pi_2(\hat{s}_1, s_2') = \left(v_2' - \frac{1}{\alpha}\right)\frac{M(\hat{s}_1, s_2')}{v_2'^2(1 + v_2'^{-1})} - F = (\alpha v_2' - 1)\frac{2(L-F)}{1 + v_2'^2} - F \qquad (16)$$

and the first order condition associated to the unconstrained profit maximization problem is

$$\frac{\partial \pi(\hat{s}_1, s_2')}{\partial v_2} = \frac{2(L-F)\left(\alpha(1 + v_2'^2) - 2v_2'(\alpha v_2' - 1)\right)}{(1 + v_2'^2)^2} = 0.$$

This simplifies to a second degree equation

$$\alpha v_2'^2 - 2v_2' - \alpha = 0$$

whose positive solution is $\bar{v}_2 = \frac{\sqrt{\alpha^2+1}+1}{\alpha}$. Since $\frac{\partial \pi(\hat{s}_1, s_2')}{\partial v_2} > 0$, for each $v_2' \in [0, \bar{v}_2)$, and $\frac{\partial \pi(\hat{s}_1, s_2')}{\partial v_2} < 0$, for each $v_2' \in (\bar{v}_2, \infty)$, we conclude that $\bar{v}_2$ corresponds to the maximum point of the monopolist 2 profit function when it adopts the industrial method and monopolist 1 plays $\hat{s}_1$. Let $s_2' = (I, \bar{v}_2)$. If s_2' is a profitable deviation, then $\pi_2(\hat{s}_1, s_2') > \pi_2(\hat{s}_1, \hat{s}_2)$ which implies that

$$(\alpha \bar{v}_2 - 1)\frac{2(L-F)}{1 + \bar{v}_2^2} - F > (\hat{v}_2 - 1)\frac{\alpha(2L-F)}{\alpha + \hat{v}_2^2}.$$

by equations (15) and (16). By substituting $\bar{v}_2$ and $\hat{v}_2$, we obtain

$$\left(\sqrt{\alpha^2+1}\right)\frac{2(L-F)}{1 + \left(\frac{\sqrt{\alpha^2+1}+1}{\alpha}\right)^2} - F > \left(\sqrt{\alpha+1}\right)\frac{\alpha(2L-F)}{\alpha + (\sqrt{\alpha+1}+1)^2},$$

which can be rewritten as

$$\frac{\alpha^2(L-F)}{\sqrt{\alpha^2+1}+1} - F > \frac{\alpha(2L-F)}{2(\sqrt{\alpha+1}+1)}.$$

With some further manipulation, we obtain

$$\alpha^2(L-F) > \left(\frac{\alpha(2L-F)}{2(\sqrt{\alpha+1}+1)} + F\right)(\sqrt{\alpha^2+1}+1),$$

which contradicts the condition in (7) and then s_2' is not a profitable deviation, a contradiction. We then conclude that $\hat{s}$ in (6) is a Nash equilibrium for $n = 2$.

We now prove the statement in *(ii)*. We proceed by contradiction and suppose that the strategy $\hat{s}$ in (6) is a Nash equilibrium for any $n > 2$. At $\hat{s}$ there is partial industrialization. Setting $q = 1$, $r = \hat{v}_n$, and $\ell = n-1$ in equation (11), we obtain

$$M(\hat{s}) = \frac{\alpha(nL - (n-1)F)\hat{v}_n^2\left(n - 1 + \hat{v}_n^{-1}\right)}{\alpha + (n-1)\hat{v}_n^2}.$$

Then, the monopolist n's profit at the strategy profile $\hat{s}$ is

$$\pi_n(\hat{s}) = (\hat{v}_n - 1)\,\frac{M(\hat{s})}{\hat{v}_n^2(n-1+\hat{v}_n^{-1})} = (\hat{v}_n - 1)\,\frac{\alpha(nL-(n-1)F)}{\alpha+(n-1)\hat{v}_n^2}.$$

We then consider the following limit

$$\begin{aligned}
\lim_{n\to\infty}\pi_n(\hat{s}) = \lim_{n\to\infty}(\hat{v}_n-1)\,\frac{\alpha(nL-(n-1)F)}{\alpha+(n-1)\hat{v}_n^2} &= \\
\lim_{n\to\infty}\left(\frac{\sqrt{(n-1)^2+(n-1)\alpha}}{n-1}\right)\frac{\alpha(nL-(n-1)F)}{\alpha+\frac{\left(n-1+\sqrt{(n-1)^2+(n-1)\alpha}\right)^2}{n-1}} &= \\
\lim_{n\to\infty}\frac{\alpha(nL-(n-1)F)\sqrt{(n-1)^2+(n-1)\alpha}}{\alpha(n-1)+\left(n-1+\sqrt{(n-1)^2+(n-1)\alpha}\right)^2} &= \frac{\alpha(L-F)}{4}
\end{aligned} \tag{17}$$

where the last equality can be found by dividing by n^2 both the numerator and the denominator. Consider the monopolist n and the strategy $s'_n = (I, v'_n)$ at which the industrial method is adopted. It is immediate to verify that the income of the economy and the monopolist n's profit at the strategy profile $(s'_n, \hat{s}_{-n})$ corresponds to those found in equations (13) and (14), respectively, for $j = n$, as all monopolist industrialize. Then, the first order condition associated to the unconstrained profit maximization problem is

$$\frac{\partial\pi_n(s'_n,\hat{s}_{-n})}{\partial v_n} = \frac{\alpha(L-F)n\left((n-1)v_n'^2+1\right) - 2(n-1)v'_n(\alpha v'_n-1)(L-F)n}{\left((n-1)v_n'^2+1\right)^2} = 0.$$

This simplifies to a second degree equation

$$(\alpha n-\alpha)v_n'^2 - 2(n-1)v'_n - \alpha = 0$$

whose positive solution is $\bar{v}_n = \frac{n-1+\sqrt{(n-1)^2+\alpha^2(n-1)}}{\alpha(n-1)}$. Since $\frac{\partial\pi_n(s'_n,\hat{s}_{-n})}{\partial v_n} > 0$, for each $v'_n \in [0,\bar{v}_n)$, and $\frac{\partial\pi_n(s'_n,\hat{s}_{-n})}{\partial v_n} < 0$, for each $v'_n \in (\bar{v}_n,\infty)$, we conclude that $\bar{v}_n$ corresponds to the maximum point of the monopolist n profit function when it adopts the industrial method and all other monopolists play $\hat{s}_{-n}$. We then define the strategy $s'_n = (I,\bar{v}_n)$. By routine calculations, we find that the profit at the maximum point is given by

$$\pi_n(s'_n,\hat{s}_{-n}) = \frac{\alpha^2(nL-nF)}{2\left(\sqrt{(n-1+\alpha^2)(n-1)}+n-1\right)} - F.$$

We next calculate the limit

$$\lim_{n\to\infty}\pi_n(s'_n,\hat{s}_{-n}) = \lim_{n\to\infty}\frac{\alpha^2(nL-nF)}{2\left(\sqrt{(n-1+\alpha^2)(n-1)}+n-1\right)} - F = \frac{\alpha^2(L-F)}{4} - F.$$

By the condition in (7), we have that

$$\frac{\alpha^2(L-F)}{4} - F > \frac{\alpha(L-F)}{4}$$

But then,

$$\lim_{n\to\infty}\pi_n(s'_n,\hat{s}_{-n}) > \lim_{n\to\infty}\pi_n(\hat{s})$$

Hence, there exists an n sufficiently large such that the monopolist n has a profitable deviation. □

Proof of Example 2. By substituting $\sigma = 2$, $L = 4.5$, $\alpha = 3$, and $F = 2$ in the inequality in (7), given by

$$\left(\frac{(2L-F)\alpha}{2(1+\sqrt{\alpha+1})} + F\right)(1+\sqrt{\alpha^2+1}) > \alpha^2(L-F) > \alpha(L-F) + 4F,$$

we obtain $22.89 > 22.5 > 15.5$. Hence, the economy satisfies the condition in (7). □

Proof of Example 3. At (s_1, s_2) there is partial industrialization. Setting $q = 1$, $r = v_2$, $n = 2$, and $\ell = 1$ in equation (11), we obtain

$$M(s_1, s_2) = \frac{\alpha(2L-F)v_2^\sigma\left(1+v_2^{1-\sigma}\right)}{\alpha + v_2^\sigma}.$$

Then, the monopolist 2's profit at the strategy profile (s_1, s_2) is

$$\pi_2(s_1, s_2) = (v_2 - 1)\,\frac{M(s_1, s_2)}{v_2^\sigma\left(1+v_2^{1-\sigma}\right)} = (v_2 - 1)\,\frac{\alpha(2L-F)}{\alpha + v_2^\sigma}$$

and the first order condition associated to the unconstrained profit maximization problem is

$$\frac{\partial\pi(s_1, s_2)}{\partial v_2} = \frac{\alpha(2L-F)\left(\alpha + \sigma v_2^{\sigma-1} + v_2^\sigma(1-\sigma)\right)}{(\alpha + v_2^\sigma)^2},$$

which is always positive as $\sigma < 1$. But then, the profit maximization problem of monopolist 2 has no solution. □

Proof of Example 4. It is immediate to check that for $b = \frac{2^\sigma 2.2 - 2.4}{2^\sigma + 2}$, the price $p^{**} = (1, 2)$ and the allocation $(x^{**}, z_1^{**}, z_2^{**})$ defined in (8) constitutes a loss-free equilibrium with partial industrialization characterized in Proposition 1. Indeed, note that $b \in [0.2, 2.2)$, for $\sigma \geq 0.98$, and this corresponds to the case in which firm 1 industrializes and firm 2 adopts the cottage method.

Next, it is straightforward to observe that the set of Pareto optimal allocations is given by the solutions to the following maximization problem

$$\begin{aligned}
&\max_{x, z_1, z_2} && \left(\frac{1}{2}x_1^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}x_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} && \\
&\text{subject to} && x_1 \leq f(z_1) && (i)\\
& && x_2 \leq f(z_2) && (ii)\\
& && z_1 + z_2 \leq 4.4, && (iii)\\
& && -x_1, -x_2 \leq 0. && (iv)
\end{aligned}$$

where constraints (i), (ii), and (iii) are binding and constraint (iv) is not binding. Since the production function is not differentiable, we deal with this maximization problem graphically. Maximum points must be tangency points between the indifference curve and the production possibility frontier $y_2(y_1)$, which is given by

$$y_2(y_1) = \begin{cases} 6.8 - 2y_1 & y_1 \leq 2, \\ 4.8 - y_1 & 2 \leq y_1 \leq 2.8, \\ 3.4 - \frac{1}{2}y_1 & 2.8 \leq y_1 \leq 6.8. \end{cases}$$

In the first case firm 1 adopts the cottage method and firm 2 industrializes, in the second case both firms industrialize, and in the third case firm 1 industrializes and firm 2 adopts the cottage method. Maximum points must satisfy the tangency condition between the production possibility frontier and the indifference curve. There are three such tangency points, one in each segment of the production possibility frontier, given by

$$(x_1', x_2') = \left(\frac{6.8}{2^\sigma+2}, \frac{2^\sigma 6.8}{2^\sigma+2}\right) \quad (x_1'', x_2'') = (2.4, 2.4) \quad (x_1''', x_2''') = \left(\frac{2^\sigma 6.8}{2^\sigma+2}, \frac{6.8}{2^\sigma+2}\right)$$

Since the commodity bundles (x_1', x_2') and (x_1''', x_2''') lead to the same utility level by symmetry of the CES utility function, we can restrict the analysis to the commodity bundles (x_1'', x_2'') and (x_1''', x_2'''). We find the value of σ for which the commodity bundle characterized by partial industrialization is higher. That is, we have to solve the following inequality

$$\left(\frac{1}{2}\left(\frac{2^\sigma 6.8}{2^\sigma+2}\right)^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}\left(\frac{6.8}{2^\sigma+2}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} > \left(\frac{1}{2}(2.4)^{\frac{\sigma-1}{\sigma}} + \frac{1}{2}(2.4)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

which simplifies in

$$\frac{17}{12}\left(2^{-\sigma}+2^{-1}\right)^{\frac{1}{\sigma-1}} > 1.$$

By taking the logarithm of both sides of the equation and by solving the equation numerically, we can conclude that the commodity bundle (x_1''', x_2''') leads to a higher utility for $\sigma > 0.9711416878379846$ and then *a fortiori* for $\sigma \geq 0.98$. Since $(x_1''', x_2''') = (x_1^{**}, x_2^{**})$, we conclude that the loss-free allocation $(x^{**}, z_1^{**}, z_2^{**})$ defined in (8) is Pareto optimal for $\sigma \geq 0.98$. □

Proof of Proposition 5. Before proving the statement of the proposition, we establish a preliminary result. Consider an allocation $(\tilde{x}, \tilde{z}_1, \ldots, \tilde{z}_n)$ which solves the following social welfare maximization problem

$$\begin{aligned}
\max_{x, z_1, \ldots, z_n} \quad & \left(\sum_{h=1}^{n} \frac{1}{n} x_h^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
\text{subject to} \quad & x_h \leq f_h(z_h) \text{ for each } h = 1, \ldots, n \qquad (i) \\
& \sum_{h=1}^{n} z_h \leq nL, \qquad (ii) \\
& -x_1, \ldots, -x_n \leq 0. \qquad (iii)
\end{aligned} \tag{18}$$

Given the allocation $(\tilde{x}, \tilde{z}_1, \ldots, \tilde{z}_n)$, we denote the set of commodities produced with the cottage method by H^C and the set of commodities produced with the industrial method by H^I. Next, consider the following maximization problem

$$\begin{aligned}
\max_{x,z_1,\ldots,z_n} \quad & \left(\sum_{h=1}^{n}\frac{1}{n}x_h^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
\text{subject to} \quad & x_h \leq z_h \text{ for each } h \in H^C \qquad (iv) \\
& x_h \leq \alpha(z_h - F) \text{ for each } h \in H^I \qquad (v) \\
& \sum_{h=1}^{n} z_h \leq nL, \qquad (vi) \\
& -x_1, \ldots, -x_n \leq 0. \qquad (vii)
\end{aligned} \tag{19}$$

By construction, the allocation $(\tilde{x}, \tilde{z}_1, \ldots, \tilde{z}_n)$ satisfies the constraints $(iv) - (vii)$. By way of contradiction, suppose that $(\tilde{x}, \tilde{z}_1, \ldots, \tilde{z}_n)$ is not a solution to the maximization problem in (19). Then, there exists another allocation $(x', z'_1, \ldots, z'_n)$ such that $u(x') > u(\tilde{x})$. Since x' satisfies constraints (iv) and (v), then it also satisfies constraint (i) by the definition of the production function in (2). But then, $(\tilde{x}, \tilde{z}_1, \ldots, \tilde{z}_n)$ cannot be a solution to the maximization problem in (18), a contradiction. Hence, if an allocation $(\tilde{x}, \tilde{z}_1, \ldots, \tilde{z}_n)$ is the solution to the maximization problem in (18), then it is also the solution to the maximization problem in (19).

We now prove the statement of the proposition by contradiction. Suppose that $(\tilde{x}, \tilde{z}_1, \ldots, \tilde{z}_n)$ is a Pareto optimal allocation and that there are two consumption goods i and l that are produced with the same technology and such that $\tilde{x}_i \neq \tilde{x}_l$. Since $\tilde{x}$ is Pareto optimal, it solves the maximization problem in (18) and the maximization problem in (19) by the preliminary result just proved. The advantage of this latter maximization problem is that both the objective function and the constraints are differentiable. The Lagrangian associated to this maximization problem is

$$\mathcal{L} = \left(\sum_{h=1}^{n}\frac{1}{n}x_h^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \sum_{h\in H^C}\lambda_h\,(x_h - z_h) - \sum_{h\in H^I}\lambda_h\,(x_h - \alpha(z_h - F)) - \lambda_{n+1}\left(\sum_{h=1}^{n} z_h - nL\right)$$

with λ_h the multipliers associated to the constraints (iv) and (v) and λ_{n+1} the multiplier associated to the constraint (vi). Note that only constraints (iv), (v), and (vi) are binding at the maximum point. The constraint qualifications are satisfied because all the constraints are linear. Then, by the Kuhn-Tucker theorem, the necessary first order conditions for a maximum are

$$\frac{\partial \mathcal{L}}{\partial x_h} = \frac{\sigma}{\sigma-1}\left(\sum_{h=1}^{n}\frac{1}{n}x_h^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}\left(\frac{1}{n}\frac{\sigma-1}{\sigma}x_h^{-\frac{1}{\sigma}}\right) - \lambda_h = 0 \text{ for } h = 1, \ldots, n \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial z_h} = \lambda_h - \lambda_{n+1} = 0 \text{ for } h \in H^C \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial z_h} = \alpha\lambda_h - \lambda_{n+1} = 0 \text{ for } h \in H^I. \tag{22}$$

Suppose first that the two commodities i and l are produced with the cottage method. Since $\tilde{x}_i \neq \tilde{x}_l$, it follows that the corresponding multipliers are such that $\lambda_i \neq \lambda_l$ by equation (20). But by equation (21), we also have that $\lambda_i - \lambda_{n+1} = \lambda_l - \lambda_{n+1}$, a contradiction. Suppose now that the two commodities i and l are produced with the industrial method. By applying the same argument, *mutatis mutandis*, a contradiction arises. Therefore, we can conclude that if an allocation $(\tilde{x}, \tilde{z}_1, \ldots, \tilde{z}_n)$ is Pareto optimal, then $\tilde{x}_i = \tilde{x}_l$ for any pair of consumption goods i and l that are produced with the same technology. $\square$

Proof of Proposition 6. Consider the loss-free allocations $(x^*, z_1^*, \dots, z_n^*)$ and $(x^{**}, z_1^{**}, \dots, z_n^{**})$ defined in Proposition 1. We define the function

$$U_1(L) = \left(\sum_{h=1}^{n} \frac{1}{n} {x_h^*}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \left(\sum_{h=1}^{n} \frac{1}{n} \left(\alpha\left(L-F\right)\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \alpha\left(L-F\right)$$

which associates to any $L > \frac{\alpha}{\alpha-1}F$ the utility level given by the loss-free allocation with full industrialization characterized in Proposition 1 for $b = 0$. Next, we define the function

$$\begin{aligned} U_2(L) = \left(\sum_{h=1}^{n} \frac{1}{n} {x_h^{**}}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} &= \\ \left(\sum_{h=1}^{\ell} \frac{1}{n} \left(\alpha\left(L+\frac{b}{\ell}-F\right)\right)^{\frac{\sigma-1}{\sigma}} + \sum_{h=\ell+1}^{n} \frac{1}{n} \left(L-\frac{b}{n-\ell}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} &= \qquad (23) \\ \left(\frac{\ell}{n} \left(\alpha\left(L+\frac{b}{\ell}-F\right)\right)^{\frac{\sigma-1}{\sigma}} + \frac{n-\ell}{n} \left(L-\frac{b}{n-\ell}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} & \end{aligned}$$

with $b \in [(n-\ell)(L-\frac{\alpha}{\alpha-1}F), (n-\ell)L)$ and $\ell = 1, \dots, n-1$, which associates to any $L > \frac{\alpha}{\alpha-1}F$ the utility level given by the loss-free allocation with partial industrialization, where ℓ firms industrialize and $n-\ell$ firms adopt the cottage method, characterized in Proposition 1. We finally define the following function

$$U_3(L) = \left(\frac{\ell}{n} \left(\alpha\left(L+\frac{(n-\ell)L}{\ell}-F\right)\right)^{\frac{\sigma-1}{\sigma}} + \frac{n-\ell}{n} \left(\frac{\alpha}{\alpha-1}F\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

Since the last $n-\ell$ firms adopt the cottage method, it follows that $L - \frac{b}{n-\ell} \leq \frac{\alpha}{\alpha-1}F$. Since it also holds that $b < L(n-\ell)$, we conclude that $U_3(L) > U_2(L)$ for any $L > \frac{\alpha}{\alpha-1}F$. Next, we consider $\lim_{L\to\infty} \frac{U_3(L)}{U_1(L)}$ which can be written as

$$\lim_{L\to\infty} \left(\frac{\ell}{n} \left(\frac{\alpha(\ell L + (n-\ell)L - \ell F)}{\alpha\ell\left(L-F\right)}\right)^{\frac{\sigma-1}{\sigma}} + \frac{n-\ell}{n} \left(\frac{\alpha F}{(\alpha-1)\alpha\left(L-F\right)}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

To solve the limit, it is convenient to further rewrite it as

$$\lim_{L\to\infty} \left(\frac{\ell}{n} \left(\frac{n-\ell\frac{F}{L}}{\ell-\ell\frac{F}{L}}\right)^{\frac{\sigma-1}{\sigma}} + \frac{n-\ell}{n} \left(\frac{\frac{F}{L}}{(\alpha-1)\left(1-\frac{F}{L}\right)}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

For the first term in the summation, we can easily check that $\lim_{L\to\infty} \frac{\ell}{n} \left(\frac{n-\ell F/L}{\ell-\ell F/L}\right)^{\frac{\sigma-1}{\sigma}} = \left(\frac{n}{\ell}\right)^{-\frac{1}{\sigma}}$. For the second term in the summation, two cases arise. If $0 < \sigma < 1$, it follows that

$$\lim_{L\to\infty} \frac{n-\ell}{n} \left(\frac{\frac{F}{L}}{(\alpha-1)\left(1-\frac{F}{L}\right)}\right)^{\frac{\sigma-1}{\sigma}} = \frac{n-\ell}{n} \left(\frac{0}{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}} = \infty.$$

But then, $\lim_{L\to\infty}\frac{U_3(L)}{U_1(L)}=0$. If $1<\sigma<\infty$, the second term in the summation is equal to zero. Then,

$$\lim_{L\to\infty}\frac{U_3(L)}{U_1(L)}=\left(\left(\frac{n}{\ell}\right)^{-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}=\left(\frac{\ell}{n}\right)^{\frac{1}{\sigma-1}}<1,$$

as $\ell<n$. For $\sigma=1$, we obtain that

$$\lim_{L\to\infty}\frac{U_3(L)}{U_1(L)}=\lim_{L\to\infty}\frac{\frac{\ell}{n}\ln\left(\alpha\left(L+\frac{(n-\ell)L}{\ell}-F\right)\right)+\frac{n-\ell}{n}\ln\frac{\alpha}{\alpha-1}F}{\ln\left(\alpha\left(L-F\right)\right)}=\frac{\ell}{n}<1$$

by applying the L'Hôpital's rule. Hence, for any $0<\sigma<\infty$, it follows that $U_1(L)>U_3(L)$, for L sufficiently high. Put differently, we can pick a $\tilde{L}>\frac{\alpha}{\alpha-1}F$ such that $U_1(L)>U_3(L)$ for each $L\geq\tilde{L}$. Since $U_3(L)>U_2(L)$ for each $L>\frac{\alpha}{\alpha-1}F$, we conclude that $U_1(L)>U_2(L)$ for each $L\geq\tilde{L}$. Therefore, the statement of the proposition follows straightforwardly. $\square$

Proof of Example 5. Let $n=2$, $\sigma=2$, $L=4.5$, $\alpha=3$, and $F=2$. By the result in Proposition 4(i), it is immediate to conclude that $\hat{s}=((I,1),(C,3))$ is a Nash equilibrium for $\Gamma_1(2)$ and that

$$(x(\hat{s}),z_1(\hat{s}),z_2(\hat{s}))=\left(\left(\frac{63}{4},\frac{7}{4}\right),\frac{29}{4},\frac{7}{4}\right)$$

is a Nash allocation. Furthermore, by the result in Proposition 1(i), with $q=r=1$, $\ell=1$ and $b=0$, it is immediate to conclude that

$$((x_1^*,x_2^*),z_1^*,z_2^*)=((7.5,7.5),4.5,4.5)$$

is a loss-free allocation. Finally, we have that $u(x(\hat{s}))\approx 7$ and that $u(x^*)\approx 7.5$. Hence, the Nash allocation is Pareto dominated by the loss-free allocation. $\square$

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